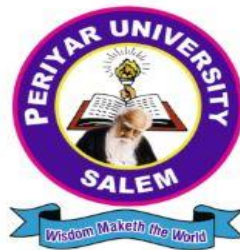


PERIYAR UNIVERSITY

**NAAC 'A++' Grade with CGPA 3.61 (Cycle - 3)
State University - NIRF Rank 56 - State Public University Rank 25
Salem-636011, Tamilnadu, India.**

CENTRE FOR DISTANCE AND ONLINE EDUCATION (CDOE)

BACHELOR OF BUSINESS ADMINISTRATION SEMESTER – VI



PROFESSIONAL COMPETENCY ENHANCEMENT: QUANTATIVE APTITUDE I (Candidates admitted from 2024 onwards)

PERIYAR UNIVERSITY

CENTRE FOR DISTANCE AND ONLINE EDUCATION (CDOE)

B.B.A 2024 admission onwards

Professional Competency Enhancement: Quantative Aptitude I

Prepared by:

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TABLE OF CONTENTS		
UNIT	TOPICS	PAGE
Syllabus		
1	Numerical computation - Number system, Ratio Proportion	06
2	Numerical Estimation - Time and Work - Time and Distance	17
3	percentages, Simple interest - Compound Interest	24
4	Data interpretation related to Averages	39
5	Application to Geometry	54

SYLLABUS

QUANTATIVE APTITUDE I

Unit -I: Numerical computation - Number system – Types of number – Application – Importance – Problems - Chain rule – problem – Ratio Proportion – Solved problems.

Unit -II: Introduction to Numerical Estimation - Time and Work - Time and Distance – Formulae – Solved Examples.

Unit -III: Introduction to percentages, Profit Loss - Discount, Simple interest - Compound Interest Partnerships, Shares - dividends – Examples

Unit -IV: Data interpretation related to Averages - Mixtures and allegations - Bar charts - Pie charts - Venn diagrams – Examples.

Unit -V: Application to Geometry - Mensuration – Solved problems.

QUANTATIVE APTITUDE I

UNIT OBJECTIVES

In this unit, learners will gain a comprehensive understanding of quantitative aptitude, including its core principles, methodologies, and applications in various contexts. Learners will be able to analyze different quantitative problems, understand and apply a range of mathematical and statistical techniques, and evaluate the problem-solving strategies needed to tackle complex quantitative challenges effectively.

QUANTITATIVE APTITUDE I- AN INTRODUCTION

Quantitative aptitude is a fundamental skill that involves the ability to understand and work with numerical data. It encompasses a broad range of mathematical concepts and techniques used to solve problems related to arithmetic, algebra, geometry, data interpretation, and logical reasoning. Mastery of quantitative aptitude is essential in various fields, including finance, engineering, science, and technology, as it enables individuals to make informed decisions based on quantitative data. This unit will provide learners with the knowledge and tools necessary to approach quantitative problems with confidence and precision, enhancing their analytical and critical thinking abilities. By the end of this unit, learners will be proficient in applying quantitative techniques to real-world scenarios, ensuring they are well-prepared for academic, professional, and everyday challenges that require strong numerical skills.

MEANING AND DEFINITION

Quantitative aptitude refers to the ability to comprehend and work with numbers and mathematical concepts efficiently. It involves skills such as performing arithmetic operations, understanding and applying mathematical principles, interpreting data, and solving quantitative problems. This aptitude is essential for making data-driven decisions and solving numerical challenges in both academic and professional contexts. Quantitative aptitude is the capacity to quickly and accurately perform mathematical calculations, analyze numerical data, and solve problems involving arithmetic, algebra, geometry, and data interpretation.

UNIT-1

NUMERICAL COMPUTATION

1.1. NUMERICAL COMPUTATION

For aptitude exams that include numerical computation, it's important to have a solid understanding of fundamental mathematical concepts and techniques. Here are some key notes tailored for such exams:

Basic Arithmetic

Be proficient in addition, subtraction, multiplication, and division of whole numbers, fractions, and decimals. Understand the order of operations (PEMDAS/BODMAS) for performing calculations involving multiple operations.

Percentage

Know how to calculate percentages, percentage changes, and percentage discounts. Understand concepts like percentage increase/decrease and compound interest.

Ratio and Proportion

Understand the relationship between ratios, fractions, and proportions. Solve problems involving direct and inverse proportions.

Averages

Understand how to calculate arithmetic mean, median, and mode. Solve problems related to average speed, average weight, average age, etc.

Basic Algebra

Solve linear equations and inequalities. Understand basic concepts like variables, constants, coefficients, and exponents.

Basic Geometry

Know geometric shapes, their properties, and formulas for calculating perimeter, area, and volume. Understand concepts like triangles, circles, rectangles, squares, and cubes.

Number Systems

Understand different number systems such as natural numbers, whole numbers, integers, rational numbers, and irrational numbers. Convert numbers between different bases (binary, octal, decimal, and hexadecimal).

Approximation

Learn techniques for approximating calculations mentally or using rounding methods. Understand significant figures and decimal places.

Data Interpretation

Be able to interpret data presented in various forms such as tables, charts, and graphs. Extract relevant information and perform calculations based on the given data.

Practice

Regular practice is key to improving numerical computation skills. Solve a variety of problems from practice books, online resources, and previous exam papers.

Remember to manage your time effectively during the exam and practice solving problems under timed conditions to improve your speed and accuracy. Understanding the underlying concepts and mastering problem-solving techniques will greatly enhance your performance in numerical computation sections of aptitude exams.

Numerical computation refers to the use of mathematical methods to perform calculations and solve numerical problems, often using computers or calculators. It encompasses a wide range of mathematical operations including basic arithmetic (addition, subtraction, multiplication, division), more complex operations involving fractions, decimals, percentages, ratios, proportions, algebra, and analysis of number series. Numerical computation is essential in various fields such as engineering, physics, finance, and computer science, where precise and efficient calculations are necessary.

Here's a concise definition:

Numerical Computation: The process of performing mathematical calculations, often with the aid of computers or calculators, to solve numerical problems involving arithmetic operations, algebra, fractions, decimals, percentages, ratios, and number series. It is widely used in various scientific, engineering, and financial applications. Numerical computation is essential in various fields such as science, engineering, finance, and computer science, where precise and efficient numerical problem-solving is required.

Applications Based on Numbers involve practical problems that test one's ability to apply mathematical concepts to real-world scenarios. These applications can include financial calculations like interest rates and profit margins, measurements of time, speed, and distance, and proportional reasoning, among others.

Numbers are mathematical objects used to count, measure, and label. They are fundamental elements in mathematics and have various types, each with specific properties and uses. Here are some key types of numbers:

1.1.1. Types of Numbers

1. Natural Numbers:

- The set of positive integers starting from 1, 2, 3, and so on.
- Used for counting.
- Example: 1, 2, 3, ...

2. Whole Numbers:

- Natural numbers including zero.
- Example: 0, 1, 2, 3, ...

3. Integers:

- Whole numbers including positive numbers, negative numbers, and zero.
- Example: -3, -2, -1, 0, 1, 2, 3, ...

4. Rational Numbers:

- Numbers that can be expressed as the quotient or fraction of two integers, where the denominator is not zero.

- Example: $1/2$, $-3/4$, 5, 0.75

5. Irrational Numbers:

- Numbers that cannot be expressed as a simple fraction. They have non-repeating, non-terminating decimal expansions.

- Example: $\sqrt{2}$, π (pi), e (Euler's number)

6. Real Numbers:

- All rational and irrational numbers.

- Example: 3, -1.5, $\sqrt{2}$, π

7. Complex Numbers:

- Numbers that have both a real part and an imaginary part.

- Example: $3 + 4i$, where i is the imaginary unit with the property $i^2 = -1$.

Applications of Numbers**1. Counting and Ordering:**

- Natural numbers are used for counting objects.

- Integers are used to order or rank items, including positions below zero.

2. Measurements:

- Rational numbers and decimals are used in measuring quantities like length, weight, and volume.

3. Finance:

- Numbers are essential in financial calculations like interest rates, profit and loss, discounts, and budgeting.

4. Science and Engineering:

- Numbers are used in formulas, equations, and models to describe physical phenomena, from the motion of planets to electrical currents.

5. Statistics and Data Analysis:

- Numbers summarize data through measures like mean, median, and standard deviation.

6. Everyday Life:

- Numbers are used in daily activities like telling time, cooking, shopping, and planning travel.

Importance of Numbers

Numbers are crucial for:

Quantification: Assigning numerical values to quantities.

Comparison: Comparing different quantities using numerical values.

Calculation: Performing arithmetic and complex mathematical operations.

Communication: Expressing measurements, quantities, and scientific concepts clearly and precisely.

Understanding the different types of numbers and their applications is fundamental to mastering mathematical concepts and solving practical problems efficiently.

Importance Facts

1. All-natural numbers are whole numbers.
2. All Whole numbers are not natural numbers.
0 is a whole number which is not a natural number.
3. Even number + Even number = Even number

Odd number + Odd number = Even number

Even number + Odd number = Odd number

Even number - Even number = Even number

Odd number – odd number = Even number

Even number - Odd number = Odd number

Odd number- Even number= Odd number

Even number x Even number = Even number

Odd number x odd number = Odd number

Even number x Odd number = Even number

4. The smallest Prime number is 2
5. The only even prime number is 2
6. The first odd prime number is 3
7. 1 is a unique number – neither prime nor composite
8. The least composite number is 4
9. The least odd composite number is 9

Let Us Sum Up

For aptitude exams, a solid grasp of fundamental mathematical concepts and techniques is essential. Proficiency in basic arithmetic, including operations with whole numbers, fractions, and decimals, along with understanding PEMDAS/BODMAS, is crucial. Additionally, knowledge of percentages, including calculating changes and discounts, and compound interest, is important. Understanding ratios and proportions, and their application in various problems, is also necessary. Basic algebra skills for solving linear equations and inequalities are required, as well as familiarity with geometric shapes and their properties for perimeter, area, and volume calculations. Mastery of number systems and conversions between bases, approximation techniques, and data interpretation are vital. Regular practice and time management during exams improve speed and accuracy. Numbers play a key role in counting, measuring, and financial calculations, with different types including natural, whole, integers, rational, irrational, real, and complex numbers. Understanding these types and their applications in real-world scenarios enhances problem-solving efficiency.

Check your Progress

1. the sum of the greatest and smallest number of five digits is
 - a) 11,110
 - b) 10,999
 - c) 109,999
 - d) 111,110

2. what is the sum of the squares of the digits from 1 to 9?
 - a) 105
 - b) 260
 - c) 285
 - d) 385

3. if $(n-1)$ is an odd number, what are the two other odd numbers nearest to it?
 - a) $n, n-1$
 - b) $n, n-2$
 - c) $n-3, n+1$
 - d) $n-3, n+5$

4. If n is a negative number, then which of the following is the least?
 - a) 0
 - b) $-n$
 - c) $2n$
 - d) n^2

5. the difference between the square of any two consecutive integers is equal to
 - a) sum of two numbers
 - b) difference of two numbers
 - c) an even number
 - d) product of two numbers

1.2. CHAIN RULE

Definition: The chain rule in quantitative aptitude is a mathematical method used to solve problems that involve finding the relationship between two or more variables that are interconnected through a series of proportional relationships. It allows for the determination of the overall effect of changes in one variable on another by sequentially applying the given ratios or rates of change.

The chain rule simplifies complex problems by breaking them down into a sequence of simpler steps, each involving a direct proportional relationship, and then combining these steps to find the final relationship between the initial and final quantities.

Direct Proportion: Two quantities are said to be directly proportional, if on the increase (or decrease) of the one, the other increases (or decreases) to the same extent.

Ex. 1. Cost is directly proportionally to the number of articles. (more Articles, More cost)

Ex. 2. Work done is directly proportional to the number of the number of men working on it. (More men, More work)

Indirect Proportion: Two quantities are said to be indirectly proportional, if on the increase of the one,

Ex. 1. The time taken by a car in covering a certain distance is inversely proportional to the speed of the car. (More speed, less is the time taken to cover a distance)

Ex. 2. Time taken to finish a work is inversely proportional to the number of persons working at it. (More persons, less is the time taken to finish a job)

Remark: In solving questions by chain rule, we compare every item with the term to be found out.

Solved Problems

Ex. 1. A canteen requires 105 kgs of wheat for a week. How many kgs of wheat will it require for 58 days?

Sol. Let the required quantity be x kg. Then,

More days, More cost (Directly Proportion)

7 : 58 :: 105 : x

$$7 \times X = 58 \times 105$$

$$x = (580105/7)$$

$$x = 870$$

Hence, the canteen will require 870 kg of wheat for 58 days.

Ex. 2. If 36 men can do a piece of work in 25 hours, in how many hours will 15 men do it?

Sol. Let the required number of hours be x. then, Less men, More hours (Indirect Proportion)

$$15 : 36 :: 25 : x$$

$$(15 * x) = (36 * 25)$$

$$x = 36 * 25 / 15$$

$$x = 60$$

Hence, 15 men can do it in 60 hours.

Ex. 3. 35 women can do a piece of work in 15 days. How many women would be required to do the same work in 25 days?

Sol. Let the required number of women be x. Then,

More days, Less Women (indirect Proportion)

$$25 : 15 :: 35 : x$$

$$(25 * x) = (15 * 35)$$

$$x = 15 * 35 / 25$$

$$x = 21$$

Hence, 21 women can do the work in 25 days.

Ex. 4. A certain number of people were supposed to complete a work in 24 days. The work, however, took 32 days since 9 people were absent throughout. How many people were supposed to be working originally?

Sol. Originally, let there be x people.

Less people, More days (Indirect Proportion)

$$(x - 9) : x :: 24 : 32$$

$$(x - 9) * 32 = x * 24$$

$$8x = 288$$

$$x = 36$$

Hence, 36 people were supposed to be working originally.

Ex. 5. If 5 students utilize 18 pencils in 9 days, how long, at the same rate, will 66 pencils last for 15 students?

Sol. Let the required number of days be x.

More students, Less days (Indirect Proportion)

More pencils, More days (Direct proportion)

$$\{ \text{Students } 15 : 5$$

$$\text{Pencil } 18 : 66 \} :: 56 : x$$

$$(20 * 6 * x) = (35 * 3 * 56)$$

$$x = (35 * 3 * 56) / 120 = 49$$

Hence the required length is 49 m.

Let Us Sum Up

The chain rule in quantitative aptitude is a method used to analyze problems involving interconnected variables through proportional relationships. It simplifies complex problems by breaking them into simpler, sequential steps and combining these to determine the final relationship between variables. Direct proportion means that as one quantity increases or decreases, the other does the same, such as the cost of articles with the number of articles or work done with the number of workers. In contrast, indirect proportion means that as one quantity increases, the other decreases, like the time taken by a car depending on speed or the time needed for work based on the number of workers. Solving chain rule problems involves comparing items with the term to be determined. For example, to find how many kilograms of wheat are needed for 58 days given 105 kg for a week, or how

long 15 men will take to complete a job compared to 36 men in 25 hours, one applies direct and indirect proportionality principles. Similarly, if 35 women can complete a task in 15 days, one can determine how many are needed to do it in 25 days by using indirect proportion. Problems may also involve calculating the original number of workers needed or how long resources will last for a different number of people.

Check your Progress

1. If 12 men can complete a job in 18 days, how many men are needed to complete the same job in 9 days?

- a) 24 men
- b) 18 men
- c) 36 men
- d) 12 men

2. A mixture contains 30 liters of liquid A and 20 liters of liquid B. If 10 liters of the mixture are replaced with pure liquid A, what is the new ratio of liquid A to liquid B in the mixture?

- a) 4:1
- b) 3:2
- c) 5:3
- d) 2:1

3. If 5 workers can complete a task in 8 hours, how many hours will it take for 10 workers to complete the same task, assuming all workers work at the same rate?

- a) 4 hours
- b) 8 hours
- c) 10 hours
- d) 16 hours

4. A factory produces 240 units of product in 6 hours. How many hours will it take to produce 400 units at the same rate?

- a) 10 hours
- b) 8 hours
- c) 12 hours
- d) 16 hours

5. If the ratio of the number of red balls to blue balls in a box is 3:5, and there are 40 blue balls, how many red balls are there in the box?

- a) 24
- b) 30

- c) 36
- d) 20

1.3. RATIO PROPORTION

1. Ratio: The ratio of two quantities a and b in the same units, is the fraction a/b and we write it as $a : b$. In the ratio $a : b$, we call a as the first term or antecedent and b, the second term or consequent.

Example. The ratio 5 : 9 represents $5/9$ with antecedent = 5, consequent = 9.

Rule: The multiplication or division of each term of a ratio by the same non-zero number does not affect the ratio.

Example. $4 : 5 = 8 : 10 = 12 : 15$ etc. Also, $4 : 6 = 2 : 3$.

2. Proportion: the equality of two ratios is called proportion.

If $a : b = c : d$, we write, $a : b :: c : d$ and we say that a, b, c, d are in proportion.

Here a and d are called extremes, while b and c are called mean terms.

Product of means = Product of extremes.

Thus, $a : b :: c : d$

$$(b \cdot c) = (a \cdot d)$$

3. (i) Fourth Proportional: If $a : b = c : d$, then d is called the fourth proportional to a, b, c.

(ii) Third Proportional: If $a : b = b : c$, then c is called the third proportional to a and b

(iii) Mean Proportional: Mean proportional between a and b is \sqrt{ab} .

4. (i) Comparison of Ratios: We say that $(a : b) > (c : d)$

$$a/b > c/d.$$

(ii) Compounded Ratio: The compounded ratio of the ratios $(a : b)$, $(c : d)$, $(e : f)$ is $(ace : bdf)$.

5. (i) Duplicate ratio of (a: b) is ($a^2 : b^2$)

(ii) Sub-duplicate ratio of (a: b) is ($\sqrt{a} : \sqrt{b}$).

(iii) Triplicate ratio of (a : b) is ($a^3 : b^3$)

(iv) Sub-triplicate ratio of (a: b) is ($\sqrt[3]{a} : \sqrt[3]{b}$)

(v) $a/b = c/d$, then $(a + b)/(a - b) = (c + d)/(c - d)$ (componendo and dividendo)

6. Variation:

(i) We say that x is directly proportional to y, if $x = ky$ for some constant k and we write, $x \propto y$

(ii) We say that x is inversely proportional to y, if $xy = k$ for some constant k and we write, $x \propto 1/y$.

7. Suppose a container contains x units of liquid from which y units are taken out and replaced by water. After n operations, the quantity of pure liquid in the final mixture = $[x(1-y/x)^n]$ units.

Solved Problems

Ex. 1. If 10% of x is equal to 20 % of y, then find x : y.

Sol. 10 % of x = 20 % of y

$$(10/100) x = (20/100) y$$

$$x/10 = y/5$$

$$x/y = 10/5 = 2.$$

Hence, x : y = 2 :1.

Ex. 2. A man spends Rs. 500 in buying 12 tables and chairs. The cost of one table is Rs. 50 and that of one chair is Rs. 40. What is the ration of the numbers of the chairs and tables purchased?

Sol. Let the number of tables purchased be x.

Then, number of chairs purchased = $(12-x)$

$$50x + 40(12-x) = 500$$

$$10x = 20$$

$$X = 2$$

So, number of tables = 2 and number of chairs = 10.

Hence, required ratio = $10 : 2 = 5 : 1$.

Ex. 3. If $7 : x = 17.5 : 22.5$, then find the value of x .

$$\text{Sol. } 7 : x = 17.5 : 22.5$$

$$17.5x = 7 \cdot 22.5$$

$$X = 7 \cdot 22.5 / 17.5 = 9$$

Hence, $x = 9$.

Ex. 4. If 20% of $(P + Q) = 50\%$ of $(P - Q)$, then find $P : Q$

$$\text{Sol. } 20\% \text{ of } (P + Q) = 50\% \text{ of } (P - Q)$$

$$2(P+Q) = 5(P-Q)$$

$$3P = 7Q$$

$$P : Q = 7 : 3.$$

Ex. 5. If 30% of $A = 0.25$ of $B = 1/5$ of C . then find $A : B : C$.

$$\text{Sol. } 30\% \text{ of } A = 0.25 \text{ of } B = 1/5 \text{ of } C$$

$$(30/100) A = (25/100) B = (1/5) C$$

$$A = 10k/3, B = 4K, C = 5K$$

$$A : B : C = 10K/3 : 4K : 5K = 10/3 : 4 : 5 = 10 : 12 : 15.$$

Let Us Sum Up

The ratio of two quantities, a and b expressed as $a:b$, represents the fraction a/b , with a being the antecedent and b the consequent. Ratios remain unchanged if both terms are multiplied or divided by the same non-zero number. Proportions, where two ratios are equal, are denoted as $a : b :: c : d$, with a and d being the extremes, and b and c the means, satisfying the product of means equals the product of extremes. Key proportional terms include the fourth proportional, third proportional, and mean proportional. Ratios can be compared directly by comparing their fractional values, and compounded ratios combine multiple ratios into a single ratio. Variations include direct proportionality, where $x = ky$, and inverse proportionality, where $xy = k$. For mixtures, the quantity of pure liquid after

several operations can be calculated using a specific formula. Example problems include finding ratios from given percentages, solving for unknowns in proportions, and determining ratios from multiple given relationships.

Check your Progress

1. If $a : b = 4 : 7$ and $b : c = 3 : 5$, what is the ratio $a : b : c$?
 - a) $12 : 21 : 35$
 - b) $12 : 21 : 30$
 - c) $8 : 14 : 25$
 - d) $16 : 28 : 35$
2. A ratio of 5:8 is equivalent to which of the following ratios?
 - a) 10:16
 - b) 15:24
 - c) 25:40
 - d) All of the above
3. In a group of students, the ratio of boys to girls is 3:4. If there are 84 students in total, how many boys are there?
 - a) 36
 - b) 42
 - c) 48
 - d) 54
4. A sum of ₹ 1250 is divided among A,B,C so that A gets $\frac{2}{9}$ of B's share and c gets $\frac{3}{4}$ of A's share. The share of C is
 - a) ₹ 75
 - b) ₹90
 - c) ₹135
 - d) ₹150
5. In a certain recipe, the ratio of sugar to flour is 2:5. If a cook uses 500 grams of flour, how much sugar is used?
 - a) 100 grams
 - b) 200 grams
 - c) 250 grams
 - d) 300 grams

Unit Summary

1. Numerical Computation and Number System

Numerical computation refers to methods and algorithms used to solve mathematical problems using numbers. The number system is a framework to represent and work with numbers. There are different types of number systems, such as binary, decimal, and hexadecimal, which are critical for computing and mathematical operations.

2. Types of Numbers

Numbers can be classified into various types based on their properties:

- Natural Numbers (N): Positive integers (1, 2, 3, ...).
- Whole Numbers (W): Natural numbers including zero (0, 1, 2, ...).
- Integers (Z): Positive and negative whole numbers (... -3, -2, -1, 0, 1, 2, 3 ...).
- Rational Numbers (Q): Numbers that can be expressed as a ratio of two integers (e.g., $\frac{1}{2}$, $\frac{3}{4}$).
- Irrational Numbers: Numbers that cannot be expressed as a ratio (e.g., $\sqrt{2}$, π).
- Real Numbers (R): All rational and irrational numbers.
- Complex Numbers (C): Numbers expressed in the form $a + bi$, where "a" and "b" are real numbers, and "i" is the imaginary unit.

3. Applications of Number Systems

- Computing: Binary (0,1) is used in computers and digital circuits.
- Finance: Decimal number systems are used in financial calculations and accounting.
- Engineering: Hexadecimal and octal systems are often used in electronic and computer system designs.

4. Importance of Number Systems

Understanding number systems is fundamental for problem-solving, computing, data analysis, and various fields of engineering, science, and business. It forms the base of mathematical reasoning and supports various operations like addition, subtraction, multiplication, and division.

Chain Rule: The Chain Rule is a technique used in mathematics to solve problems involving the relationship between three or more variables. It allows breaking down complex multiplications into simpler steps.

Glossary

1. Natural Numbers: Positive counting numbers starting from 1.
2. Integers: Whole numbers, including positive, negative numbers, and zero.
3. Rational Numbers: Numbers that can be expressed as fractions.
4. Irrational Numbers: Numbers that cannot be expressed as fractions, with non-repeating decimals.
5. Binary System: A base-2 numeral system, using only 0 and 1, used in computers.
6. Decimal System: A base-10 system, used for general calculations (0–9 digits).
7. Proportion: An equation representing the equality of two ratios.
8. Chain Rule: A mathematical rule to solve proportion problems involving three or more variables.
9. Ratio: A relationship between two numbers indicating how many times the first number contains the second.
10. Complex Numbers: Numbers in the form of $a + bi$, where "i" is the imaginary unit.

UNIT – II

NUMERICAL ESTIMATION – I

2.1. NUMERICAL ESTIMATION

Numerical estimation involves making educated guesses or approximations of quantities or values without using precise calculations. It's a valuable skill in many areas of life, from budgeting and planning to problem-solving and decision-making. For example, estimating the number of people in a crowd, the cost of a shopping trip, or the time it takes to complete a task are all common applications of numerical estimation.

2.1.1. TIME AND WORK

Time and work problems are common in aptitude tests and have numerous practical applications in various fields. Here are some key notes to help you tackle such problems effectively:

Basic Concepts

Time and work problems typically involve calculating the amount of work done by individuals or groups over a certain period of time. Work is often measured in terms of units completed, such as tasks accomplished, items produced, or distance covered. Time is the duration it takes to complete a certain amount of work, usually measured in hours, days, or any other relevant unit.

Rate of Work

The rate of work indicates how quickly an individual or a group can complete a certain task. It is usually expressed as the fraction of work completed per unit time. For example, if a person can complete $\frac{1}{5}$ th of a job in one hour, their rate of work is $\frac{1}{5}$ per hour. Rates of work can be added or combined when multiple individuals are working together.

Inverse Relationship between Time and Work

There's an inverse relationship between the time taken to complete a task and the number of people working on it. Generally, more people working on a task will result in less time required to complete it, and vice versa. This relationship forms the basis for many time and work problems. For instance, if 4 people can complete a task in 5 hours, the same task will be completed faster if more people are added to the team.

Formulas

For individual work:

$$\text{Work} = \text{Rate} \times \text{Time}$$

For combined work:

If A can do a piece of work in 'a' days and B can do the same work in 'b' days, then A and B together can do the work in

Applications

Time and work problems are commonly encountered in fields such as project management, manufacturing, construction, and logistics. They are also frequently used in competitive exams and aptitude tests to assess problem-solving skills and numerical reasoning abilities.

Strategies for Problem Solving

- Identify the rate of work for each individual or group involved.
- Determine the total work required to be done.
- Use the given information to set up equations or expressions relating to time and work.
- Solve the equations to find the unknown quantities, such as time taken or number of people needed.

Remember, practice is key to mastering time and work problems. Work through various examples to become comfortable with different problem-solving techniques and strategies.

Important facts and formulae

- If A can do a piece of work in n days, then A's 1 day's work = $1/n$.
- If A's 1 day's work = $1/n$, then A can finish the work in n days.
- If A is thrice as good a workman as B, then:

Ratio of work done by A and B = 3 : 1

Ratio of times taken by A and B to finish a work = 1 : 3.

Solved Problem

Ex. 1. If Roger can do a piece of work in 8 days and Antony can complete the same work in 5 days, in how many days will both of them together complete it?

Sol. Roger's 1 day's work = $1/8$; Antony's 1 day's work = $1/5$.

(Roger + Antony)'s 1 day's work = $(1/8 + 1/5) = 13/40$.

Both Roger and Antony will complete the work in $40/13 = 3(1/13)$ days.

Ex. 2. A and B together can complete a piece of work in 15 days and B alone in 20 days. In how many days can A alone complete the work?

Sol. (A + B)'s 1 day's work = $1/15$; B's 1 day's work = $1/20$.

A's 1 day's work = $(1/15 - 1/20) = 1/60$.

Hence, A alone can complete the work in 60 days.

Ex. 3. A alone can complete a piece of work of Rs. 300 in 6 days; but by engaging as assistant, the work is completed in 4 days. Find the share to be received by the assistant.

Sol. Assistant's 1 day's work = $1/4 - 1/6 = 1/12$.

A's share: Assistant's share = Ratio of their 1 day's work = $1/6 : 1/12 = 2 : 1$.

Hence, assistant's share = Rs. $(300 \times 1/3) =$ Rs. 100.

Ex. 4. A can do a work in 4 days, B in 5 days and C in 10 days. Find the time taken by A, B and C to do the work together.

Sol. A's 1 day's work = $1/4$; B's 1 day's work = $1/5$; C's day's work = $1/10$.

(A + B + C)'s 1 day's work = $(1/4 + 1/5 + 1/10) = 11/20$.

Hence, A, B and C together can do the work in $20/11 = \left(1 \frac{9}{11}\right)$ days.

Ex. 5. A and B undertake to do piece of work for Rs. 600. A alone can do it in 6 days while B alone can do it in 8 days. With the help of C, they finish it in 3 days. Find the share of each.

Sol. C's 1 day's work = $1/3 - (1/6 + 1/8) = 1/24$.

A : B : C = Ratio of their 1 day's work $1/6 : 1/8 : 1/24 = 4 : 3 : 1$.

A's share = Rs. $\left(600 \times \frac{4}{8}\right) = \text{Rs. } 300$, B's share = Rs. $\left(600 \times \frac{3}{8}\right) = \text{Rs. } 225$.

C's share = Rs. $[600 - (300 + 225)] = \text{Rs. } 75$.

2.1.2. TIME AND DISTANCE

Time and distance problems involve calculating the time taken to travel a certain distance at a given speed, or calculating the speed of travel based on the time taken to cover a certain distance. Here are some key notes to help you understand and solve time and distance problems:

Basic Concepts

Time and distance problems deal with the relationship between the time taken to travel a certain distance and the speed at which the travel occurs. Speed is the rate of motion, typically measured in units such as kilometres per hour (km/h) or meters per second (m/s).

Distance is the amount of space between two points, usually measured in units like kilometres (km) or meters (m). Time is the duration it takes to travel a certain distance, usually measured in hours, minutes, or seconds.

Speed Formula

$$\text{Speed} = \text{Distance} / \text{Time}$$

This formula can be rearranged to find the time taken or the distance covered, depending on the known quantities.

Uniform Speed

When an object travels at a constant speed, the distance covered is directly proportional to the time taken. This means that if the speed remains constant, you can use the formula $\text{Speed} = \text{Distance} / \text{Time}$ to solve for unknowns.

Relative Speed

When two objects are moving in the same direction, their relative speed is the difference between their individual speeds. When two objects are moving towards each other, their relative speed is the sum of their individual speeds. These concepts are often used in problems involving overtaking, meeting, or crossing situations.

Applications

Time and distance problems are applicable in various real-life scenarios, such as calculating travel times for commuting, estimating arrival times for deliveries, or determining the speed of vehicles. They are also commonly encountered in competitive exams, aptitude tests, and job interviews to assess problem-solving skills and numerical reasoning abilities.

Strategies for Problem Solving

- Identify the known quantities (distance, speed, or time) and the unknown quantity to be found.
- Choose an appropriate formula or approach based on the given information and the type of problem (e.g., constant speed, relative speed).
- Substitute the known values into the formula and solve for the unknown quantity.
- Pay attention to units and conversions if necessary.

Important Facts and Formulae

1. $\text{Speed} = (\text{distance} / \text{time})$, $\text{Time} = (\text{Distance} / \text{Speed})$, $\text{Distance} = (\text{Speed} \times \text{Time})$

2. $x \text{ km/hr} = (x * \frac{5}{18}) \text{ m/sec}$

3. $x \text{ m/sec} = (x * \frac{18}{5}) \text{ km/hr}$

4. If the ratio of the speeds of A and B is $a : b$, then the ratio of the times taken by them to cover the same distance is $1/a : 1/b$ or $b : a$.
5. Suppose a man covers a certain distance at x km/hr and an equal distance at y km/hr. Then, the average speed during the whole journey is $(2xy / x + y)$ km/hr.
6. Suppose two men are moving in the same direction at u m/s and v m/s respectively, where $u > v$, then their relative speed = $(u - v)$ m/s.
7. Suppose two men are moving in opposite directions at u m/s and v m/s respectively, then their relative speed = $(u + v)$ m/s.
8. If two persons A and B start at the same time in opposite directions from two points and after passing each other they complete the journeys in a and b hours respectively, then A's speed : B's speed = $\sqrt{b} : \sqrt{a}$.

Solved Problems

Ex. 1. A train travels 82.6 km/hr. How many metres will it travel in 15 minutes?

Sol. Distance travelled in 1 min = $(82.6 / 60)$ km.

Distance travelled in 15 min = $(82.6 / 60) \times 15$ km = 20650 m.

Ex. 2 How many minutes does Aditya take to cover a distance of 400 m, if he runs at a speed of 20 km/hr?

Sol. Aditya's speed = $20 \text{ km/hr} = 50/9 \text{ m/sec}$

Time taken to cover 400m = $(400 * \frac{9}{50}) \text{ sec} = (1 \frac{1}{5}) \text{ min}$.

Ex. 3. Excluding the stoppages, the speed of a bus is 64 km/hr and including the stoppages, the speed of the bus is 48 km/hr. For how many minutes does the bus stop per hour?

Sol. Due to stoppage, the bus covers $(64 - 48) = 16$ km less per hour.

Time taken to cover $(16/64) \times 60$ min = 15 min.

Ex. 4. A goods train leaves a station at a certain time and at a fixed speed. After 6 hours, an express train leaves the same station and moves in the same direction at a

uniform speed of 90 kmph. This train catches up the goods train in 4 hours. Find the speed of the goods train.

Sol. Let the speed of the goods train be x kmph.

Distance covered by goods train in 10 hours = Distance covered by express train in 4 hours

$$\therefore 10x = 4 \times 90 \text{ or } x = 36.$$

So, speed of goods train = 36 kmph.

Ex. 5. Sneha is picked up by her father by car from college every day. The college gets over at 4 p.m. daily. One day, the college got over an hour earlier than usual. Sneha started walking towards her house. Her father, unaware of this fact, leaves his house as usual, meets his daughter on the way, picks her up and they reach the house 15 minutes earlier than usual. What is the ratio of the father's driving speed to Sneha's walking speed?

Sol. Since 15 minutes are saved, it means that Sneha's father drives from the meeting point to the college and back to the meeting point in 15 min.

i.e. he can drive from the meeting point to the college in $15/2 = 7.5$ min. But he reaches the college daily at 4 p.m. So Sneha and her father meet on the way at $3 : (52\frac{1}{2})$ p.m

Thus, Sneha walked for 52.5 min and covered the same distance as covered by her father in 7.5 min. Since speed varies inversely as time taken to cover a distance, we have:

$$\text{Father's driving speed/ Sneha's walking speed} = 52.5/7.5 = 7/1$$

Hence, required ratio = 7 : 1.

Let Us Sum Up

Numerical estimation is crucial for making educated guesses or approximations in various scenarios, from budgeting to problem-solving. In time and work problems, understanding the rate of work, which is the fraction of work done per unit time, is essential. For instance, if multiple people work together, their combined

rate is the sum of their individual rates. Time and distance problems involve calculating the time taken, speed, or distance using formulas such as $\text{Speed} = \text{Distance} / \text{Time}$. Relative speed comes into play when objects move in the same or opposite directions. Real-life applications of these problems include travel time calculations and project management. Practice helps in mastering these problems and applying them efficiently in exams and daily tasks.

Check your Progress

1. If A can complete a piece of work in 12 days and B can complete the same work in 15 days, how many days will A and B together take to complete the work?
 - a) 6.5 days
 - b) 6 days
 - c) 7.5 days
 - d) 7 days

2. A train travels at a speed of 90 km/h. How many meters will it travel in 20 minutes?
 - a) 30,000 meters
 - b) 18,000 meters
 - c) 15,000 meters
 - d) 12,000 meters

3. If two cars are moving towards each other with speeds of 60 km/h and 80 km/h respectively, what is their relative speed when moving towards each other?
 - a) 140 km/h
 - b) 120 km/h
 - c) 100 km/h
 - d) 40 km/h

4. A person walks at 4 km/h and another person runs at 12 km/h. What is the ratio of their speeds?
 - a) 1:3
 - b) 1:2
 - c) 3:1
 - d) 1:4

5. If Sneha's father drives from a meeting point to her college and back in 15 minutes, and Sneha walks the same distance in 52.5 minutes, what is the ratio of the father's driving speed to Sneha's walking speed?

- a) 7:1
- b) 1:7
- c) 5:1
- d) 1:5

Unit Summary

Numerical estimation plays a vital role in solving problems related to time and work, and time and distance. It helps in making quick approximations that are useful for real-life applications, such as planning work schedules, estimating travel times, and evaluating productivity. This unit focuses on understanding key concepts, applying relevant formulae, and solving problems through estimation methods.

1. Time and Work

Problems involving time and work revolve around determining the amount of work completed by individuals or groups in a given time period. The efficiency of individuals and machines in completing tasks is essential for problem-solving in this category.

Formulae:

- Work Done = Rate of Work \times Time

Where Rate of Work refers to the portion of work completed per unit of time.

- Rate of Work = Work Done / Time Taken

2. Time and Distance

Problems involving time and distance focus on determining the relationship between speed, distance, and time. Concepts like average speed, relative speed, and uniform motion play an essential role in these calculations.

Formulae:

- Speed = Distance / Time

- Distance = Speed \times Time

- $\text{Time} = \text{Distance} / \text{Speed}$

- Relative Speed:

- When two objects move in the same direction, $\text{Relative Speed} = | \text{Speed}_1 - \text{Speed}_2 |$.

- When two objects move in opposite directions, $\text{Relative Speed} = \text{Speed}_1 + \text{Speed}_2$.

- $\text{Average Speed} = \text{Total Distance} / \text{Total Time}$

Glossary

1. Work Done: The total amount of work completed, often represented as a fraction of the total task or job.

2. Rate of Work: The rate at which a person or machine completes work, typically expressed as a fraction of the total work done per unit of time.

3. Efficiency: The capability of an individual or a machine to complete work in a given time. Higher efficiency implies completing more work in less time.

4. Speed: The rate at which an object covers distance, measured as distance per unit of time (e.g., meters per second or kilometers per hour).

5. Distance: The total length of the path traveled by an object, usually measured in meters or kilometers.

6. Time: The duration taken to cover a certain distance or complete a task, measured in seconds, minutes, or hours.

7. Relative Speed: The speed of one object as observed from another moving object. It is used in problems involving two moving objects.

8. Combined Work: The total work done when multiple people or machines work together, calculated by adding their individual rates of work.

9. Average Speed: The total distance covered divided by the total time taken, used when speed varies during a journey.

10. Inverse Proportion: A relationship where one value increases as another decreases. In time and work problems, more workers mean less time required for the same work, exemplifying an inverse relationship.

UNIT-III

NUMERICAL ESTIMATION II

3.1. PERCENTAGE

The word “percent” is derived from the Latin words “per centum”, which means “per hundred”. A percentage is a fraction with denominator hundred, it is denoted by the symbol %. Numerator of the fraction is called the rate per cent.

VALUE OF PERCENTAGE:

Value of percentage always depends on the quantity to which it refers: Consider the statement, “65% of the students in this class are boys”. From the context, it is understood that boys form 65% of the total number of students in the class. To know the value of 65%, the value of the total number of students should be known. If the total number of students is 200, then,

$$\text{The number of boys} = 200 \times 65 / 100 = 130;$$

It can also be written as $(200) \times (0.65) = 130$. Note that the expressions 6%, 63%, 72%, 155% etc. Do not have any value intrinsic to themselves. Their values depend on the quantities to which they refer. To express the fraction equivalent to %: Express the fraction with the denominator 100, then the numerator is the answer.

Solved Problems

Ex.1.

If 20% of $x = y$, what is the value of $y\%$ of 20 in terms of x ?

Sol.

Given,

$$20\% \text{ of } x = y$$

$$\Rightarrow (20/100) x = y$$

$$y\% \text{ of } 20$$

$$= (y/100) \cdot 20$$

$$= [(20x/100) / 100] \times 20$$

$$= 4x/100$$

$$= 4\% \text{ of } x$$

Ex. 2. Express each of the following as a fraction:

i) 56%

ii) 4 %

iii) 0.6 %

iv) 0.08 %

Sol.

$$\text{i) } 56\% = \frac{56}{100} = \frac{14}{25}$$

$$\text{ii) } 4\% = \frac{4}{100} = \frac{1}{25}$$

$$\text{iii) } 0.6\% = \frac{0.6}{100} = \frac{6}{1000} = \frac{3}{500}$$

$$\text{iv) } 0.08\% = \frac{0.08}{100} = \frac{1}{1250}$$

Ex.3. During one year, the population of a town increased by 5% and during the next year, the population decreased by 5%. If the total population is 9975 at the end of the second year, then what was the population size in the beginning of the first year?

Sol. Population in the beginning of the first year

$$= \frac{9975}{\left(1 + \frac{5}{100}\right)\left(1 - \frac{5}{100}\right)}$$

$$= 10000.$$

Ex.4. Ajay ordered 4 pairs of black socks and some additional pairs of blue socks. The price of the black socks per pair was twice that of the blue ones. When the order was filled, it was found that the number of pairs of the two colours had been interchanged. This increased the bill by 50%. Find the ratio of the number of pairs of black socks to the number of pairs of blue socks in the original order.

Sol. Suppose he ordered n pairs of blue socks.

Let the price of each pair of blue socks be Rs. x.

Then, price of each pair of black socks = Rs. 2x.

Actual bill = (4 × 2x + nx) = Rs. (8x + nx).

Bill made on interchange = Rs. $(2nx + 4x)$

$$\Rightarrow 2(2nx + 4x) = 3(8x + nx)$$

$$\Rightarrow 4nx + 8x = 24x + 3nx \Rightarrow nx = 16x \Rightarrow n = 16.$$

Hence, required ratio = $4:16 = 1:4$.

Ex. 5. The salary of a person was reduced by 10%. By what percent should his reduced salary be raised so as to bring it at par with his original salary?

Sol. Let the original salary be Rs. 100. New salary = Rs. 90.

Increase on 90 = 10. Increase on 100 = $(\frac{10}{90} \times 100)\% = 11\frac{1}{9}\%$.

Let Us Sum Up

Percentages represent a fraction with a denominator of 100, denoted by the symbol %. To calculate the value of a percentage, you need to know the total quantity it refers to. For example, 65% of 200 students equals 130 boys. Percentages do not have intrinsic values; their significance depends on the context they are applied to. To convert percentages to fractions, express the percentage as a fraction with 100 in the denominator, simplifying if needed. For instance, 56% converts to $\frac{56}{100}$ or $\frac{14}{25}$. When a population increases by 5% one year and then decreases by 5% the next year, the final population of 9975 can be used to determine the initial population size. For a problem involving pricing, if black socks' price is twice that of blue socks, and switching the quantities increases the bill by 50%, the original ratio of black to blue socks is 1:4. Lastly, if a salary is reduced by 10%, it needs to be increased by approximately 11.11% to return to the original amount.

Check your Progress

1. If 25% of a number is 80, what is the number?

- a) 200
- b) 320
- c) 100
- d) 3200

2. Which of the following represents 12% as a fraction in its simplest form?

- a) $\frac{3}{25}$
- b) $\frac{12}{100}$
- c) $\frac{6}{50}$
- d) $\frac{1}{8}$

3. A population of a town increased by 10% in the first year and then decreased by 10% in the second year. If the population at the end of the second year is 8100, what was the initial population?

- a) 7500
- b) 9000
- c) 10000
- d) 8250

4. Ajay's bill increased by 50% when the number of pairs of black and blue socks was interchanged. If the original number of black socks was 4 and the number of blue socks was n , what is the value of n ?

- a) 12
- b) 16
- c) 20
- d) 24

5. If a person's salary is reduced by 10%, by what percentage should the reduced salary be increased to restore it to the original amount?

- a) 10%
- b) 9%
- c) 11.11%
- d) 12%

3.2. PROFIT AND LOSS

Cost Price: The price at which an article is purchased, is called its cost price, abbreviated as C.P.

Selling Price: The price at which an article is sold, is called its selling price, abbreviated as S.P.

Profit or Gain: If S.P is greater than C.P., seller is said to have a profit or gain.

Loss: If S.P. is less than C.P., the seller is said to have incurred a loss.

$$I. \text{ Gain} = (\text{S.P}) - (\text{C.P.})$$

$$II. \text{ Loss} = (\text{C.P}) - (\text{S.P.})$$

III. Loss or gain is always reckoned on C.P

$$IV. \text{ Gain \%} = \text{Gain} \times 100 / \text{C.P}$$

$$V. \text{ Loss \%} = \text{Loss} \times 100 / \text{C.P}$$

$$VI. \text{ S.P.} = \frac{(100 + \text{Gain \%})}{100} \times \text{C.P}$$

$$VII. \text{ S.P.} = \frac{(100 + \text{LOSS \%})}{100} \times \text{C.P}$$

$$VIII. \text{ C.P.} = \frac{(100)}{(100 + \text{Gain \%})} \times \text{S.P}$$

$$IX. \text{ C.P.} = \frac{(100)}{(100 + \text{loss\%})} \times \text{S.P}$$

X. If an article is sold at a gain of say, 35%, then S.P. = 135% of C.P.

XI. If an article is sold at a loss of say, 35%, then S.P. = 65 % of C.P.

Solved Examples

Ex.1. Mansi purchased a car for Rs. 2,50,000 and sold it for Rs. 3,48,000. What is the percent profit she made on the car?

Sol. C.P. = Rs. 250000 ; S.P. = Rs. 348000.

Profit = Rs. (348000 – 250000) = Rs. 98000.

Profit % = $\left(\frac{98000}{250000} \times 100 \right) = 39.2 \%$

Ex.2. If C.P. is Rs. 2516 and S.P. is Rs. 2272, find the percentage of loss.

Sol. C.P. = Rs. 2516, S.P. = Rs. 2272. Loss = Rs. (2516 – 2272) = Rs 244.

$$\text{Loss \%} = \left(\frac{244}{2516} \times 100\right) \% = 9.69\%.$$

Ex.3. Find S.P., when

i) C.P. = Rs. 56.26, Gain = 20%

ii) C.P. = Rs. 80.40, Loss = 15%

Sol. i) S.P. = 120% of Rs. 56.25 = Rs. $\left(\frac{120}{100} \times 56.25\right)$ = Rs. 67.50.

ii) S.P. = 85% of Rs. 80.40 = Rs. $\left(\frac{85}{100} \times 80.40\right)$ = Rs. 68.34.

Let Us Sum Up

I. Gain = (S.P) – (C.P.)

II. Loss = (C.P) – (S.P.)

III. Loss or gain is always reckoned on C.P

IV. Gain % = Gain $\times 100$ / C.P

V. Loss % = Loss $\times 100$ / C.P

VI. S.P. = $\frac{(100 + \text{Gain \%})}{100} \times \text{C.P}$

VII. S.P. = $\frac{(100 + \text{LOSS \%})}{100} \times \text{C.P}$

VIII. C.P. = $\frac{(100)}{(100 + \text{Gain \%})} \times \text{S.P}$

IX. C.P. = $\frac{(100)}{(100 + \text{loss \%})} \times \text{S.P}$

Check your Progress

1. If the Cost Price (C.P.) of an article is Rs. 500 and the Selling Price (S.P.) is Rs. 600, what is the percentage gain?

a) 20%

b) 25%

- c) 30%
- d) 50%

2. An article is sold for Rs. 450, incurring a loss of 10%. What was the Cost Price (C.P.) of the article?

- a) Rs. 500
- b) Rs. 550
- c) Rs. 600
- d) Rs. 750

3. If the Cost Price of an item is Rs. 200 and it is sold at a gain of 25%, what is the Selling Price (S.P.)?

- a) Rs. 240
- b) Rs. 250
- c) Rs. 275
- d) Rs. 300

4. If an article is bought for Rs. 400 and sold for Rs. 350, what is the loss percentage?

- a) 10%
- b) 12.5%
- c) 15%
- d) 20%

5. For a gain of 15%, if the Selling Price (S.P.) of an article is Rs. 115, what was the Cost Price (C.P.)?

- a) Rs. 100
- b) Rs. 110
- c) Rs. 130
- d) Rs. 140

3.3. SIMPLE INTEREST

I. **Principal:** The money borrowed or lent out for a certain period is called the principal or the sum.

II. Interest: Extra money paid for using other's money is called interest.

III. Simple Interest (S.I.): If the interest on a sum borrowed for a certain period is reckoned uniformly, then it is called simple interest.

Let Principal = P, Rate = R% per annum (p.a.) and Time = T years.

$$\text{Then, i) S.I} = \left(\frac{P \times R \times T}{100} \right)$$

$$\text{ii) P} = \left(\frac{100 \times \text{S.I.}}{R \times T} \right)$$

$$\text{iii) R} = \left(\frac{100 \times \text{S.I.}}{P \times T} \right)$$

$$\text{iv) T} = \left(\frac{100 \times \text{S.I.}}{P \times R} \right)$$

Solved Examples

Ex.1. Find the simple interest on Rs. 68000 at $16\frac{2}{3}$ % per annum for 9 months.

Sol. P = Rs. 68000, R = $\frac{50}{3}$ % p.a and T = $\frac{9}{12}$ years = $\frac{3}{4}$ years.

$$\begin{aligned} \text{S.I} &= \left(\frac{P \times R \times T}{100} \right) \\ &= \left(68000 \times \frac{50}{3} \times \frac{3}{4} \times \frac{1}{100} \right) \\ &= \text{Rs.8500.} \end{aligned}$$

Ex. 3. A town has a population of 20,000. The population increases by 10% per year. What will be the population after 2 years?

Sol.

$$\text{Here, R} = 10/100$$

$$P = 20000$$

$$T = 2$$

$$\begin{aligned} \text{Population after 2 years will be} &= P[1 + (R/100)]^T \\ &= 20000[1 + (10/100)]^2 \\ &= 20000(1.1)^2 \\ &= 24200. \end{aligned}$$

Ex. 4. The time required for a sum of money to amount to five times itself at 16% simple interest p.a. will be:

Sol.

Let the sum of money be Rs. x and the time required to amount to five times itself be T years.

Principal amount = Rs. x

Amount after T years = Rs. $5x$

So, the interest in ' T ' year should be Rs. $(5x - x) = \text{Rs. } 4x$.

$R = 16\%$

Using simple interest formula,

$$(P \times T \times R)/100 = SI$$

Where, P = Principal amount, T = Duration in years, R = Interest rate per year, SI = Simple interest

Then,

$$(x \times T \times 16)/100 = 4x$$

$$\Rightarrow T \times (16/100) = 4$$

$$\Rightarrow T = 400/16 = 25$$

\therefore The required time = 25 years.

Ex. 5. The rate of simple interest per annum at which a sum of money doubles itself in $16\frac{2}{3}$ years is:

Solution:

Let the principal amount be P .

Now, the amount A after $16\frac{2}{3}$ years is doubled.

Hence, amount is $2P$.

$$I = P \times R \times T/100$$

Where,

P = principal amount

R = rate of interest

T = time in years = $16\frac{2}{3} = \frac{50}{3}$

I = simple interest

Amount A = I + P

According to question,

$$A = P + (P \times R \times T/100)$$

$$2P = P + (P \times R \times T/100)$$

$$P = P \times R \times T/100$$

$$R = 100/T$$

$$R = 100 \times 3/50$$

$$R = 6\%$$

Let Us Sum Up

Simple Interest (S.I.) refers to the interest calculated uniformly on a principal amount over time. The principal (P) is the initial amount borrowed or lent. The interest (I) is the additional money paid for using this amount. $S.I = \left(\frac{P \times R \times T}{100} \right)$, $P = \left(\frac{100 \times S.I}{R \times T} \right)$, $R = \left(\frac{100 \times S.I}{P \times T} \right)$, $T = \left(\frac{100 \times S.I}{P \times R} \right)$. For example, a principal of Rs. 68,000 at an interest rate of $16\frac{2}{3}\%$ per annum for 9 months yields Rs. 8,500 in interest. If a sum amounts to five times itself at 16% interest, it takes 25 years. Additionally, to double a sum in $16\frac{2}{3}$ years, the annual interest rate must be 6%.

Check your Progress

1. What is the simple interest on Rs. 10,000 at a rate of 8% per annum for 3 years?

- a) Rs. 2,400
- b) Rs. 2,800
- c) Rs. 3,200
- d) Rs. 3,600

2. If a principal amount of Rs. 5,000 earns Rs. 1,000 as simple interest in 4 years, what is the annual interest rate?

- a) 5%
- b) 10%
- c) 12%
- d) 15%

3. A sum of money doubles itself in 5 years at a certain rate of simple interest. What is the rate of interest per annum?

- a) 10%
- b) 12%
- c) 20%
- d) 25%

4. What will be the amount after 2 years if Rs. 4,000 is invested at an annual simple interest rate of 6%?

- a) Rs. 4,480
- b) Rs. 4,720
- c) Rs. 4,800
- d) Rs. 4,960

5. If Rs. 8000 amounts to Rs. 10,000 in 5 years at simple interest, what is the rate of interest per annum?

- a) 4%
- b) 5%
- c) 6%
- d) 7%

3.4. COMPOUND INTEREST

Sometimes it so happens that the borrower and the lender agree to fix up a certain unit of time, say yearly or half-yearly or quarterly to settle the previous account. In such cases, the amount after first unit of time becomes the principal for the second unit, the amount after second unit becomes the principal for the third unit and so on. After a specified period, the difference between the amount and the money borrowed is called the Compound Interest (abbreviated as C.I.) for that period.

Let Principal = P, Rate = R % per annum, Time = n years.

I. When interest is compounded Annually:

$$\text{Amount} = P \left(1 + \frac{R}{100} \right)^n$$

II. When interest is compounded Half-yearly:

$$\text{Amount} = P \left(1 + \frac{R/2}{100} \right)^{2n}$$

III. When interest is compounded Quarterly:

$$\text{Amount} = P \left(1 + \frac{R/4}{100} \right)^{4n}$$

IV. When interest is compounded annually but time is in fraction, say $3\frac{2}{5}$ years.

$$\text{Amount} = P \left(1 + \frac{R}{100} \right)^3 \times \left(1 + \frac{\frac{2}{5}R}{100} \right)$$

Solved Problems

Ex.1. Find the compound interest (CI) on Rs. 12,600 for 2 years at 10% per annum compounded annually.

Sol.

Given,

Principal (P) = Rs. 12,600

Rate (R) = 10

Number of years (n) = 2

$$A = P[1 + (R/100)]^n$$

$$= 12600[1 + (10/100)]^2$$

$$= 12600[1 + (1/10)]^2$$

$$= 12600 [(10 + 1)/10]^2$$

$$= 12600 \times (11/10) \times (11/10)$$

$$= 126 \times 121$$

$$= 15246$$

Total amount, A = Rs. 15,246

Compound interest (CI) = A – P

$$= \text{Rs. } 15,246 - \text{Rs. } 12,600$$

$$= \text{Rs. } 2646$$

Ex.2. At what rate of compound interest per annum, a sum of Rs. 1200 becomes Rs. 1348.32 in 2 years?

Sol.

Let R% be the rate of interest per annum.

Given,

Principal (P) = Rs. 1200

Total amount after 2 years (A) = Rs. 1348.32

$$n = 2$$

We know that,

$$A = P[1 + (R/100)]^n$$

$$\text{Rs. } 1348.32 = \text{Rs. } 1200[1 + (R/100)]^2$$

$$1348.32/1200 = [1 + (R/100)]^2$$

$$[1 + (R/100)]^2 = 134832/120000$$

$$[1 + (R/100)]^2 = 2809/2500$$

$$[1 + (R/100)]^2 = (53/50)^2$$

$$1 + (R/100) = 53/50$$

$$R/100 = (53/50) - 1$$

$$R/100 = (53 - 50)/50$$

$$R = 300/50$$

$$R = 6$$

Hence, the rate of interest is 6%.

Ex.3. A TV was bought for Rs. 21,000. The value of the TV was depreciated by 5% per annum. Find the value of the TV after 3 years. (Depreciation means the reduction of value due to use and age of the item)

Sol.

$$\text{Principal (P)} = \text{Rs. } 21,000$$

$$\text{Rate of depreciation (R)} = 5\%$$

$$n = 3$$

Using the formula of CI for depreciation,

$$A = P[1 - (R/100)]^n$$

$$A = \text{Rs. } 21,000[1 - (5/100)]^3$$

$$= \text{Rs. } 21,000[1 - (1/20)]^3$$

$$= \text{Rs. } 21,000[(20 - 1)/20]^3$$

$$= \text{Rs. } 21,000 \times (19/20) \times (19/20) \times (19/20)$$

$$= \text{Rs. } 18,004.875$$

Therefore, the value of the TV after 3 years = Rs. 18,004.875.

Ex. 4. Find the compound interest on Rs 48,000 for one year at 8% per annum when compounded half-yearly.

Sol.

Given,

$$\text{Principal (P)} = \text{Rs } 48,000$$

$$\text{Rate (R)} = 8\% \text{ p.a.}$$

$$\text{Time (n)} = 1 \text{ year}$$

Also, the interest is compounded half-yearly.

$$\text{So, } A = P[1 + (R/200)]^{2n}$$

$$\begin{aligned}
 &= \text{Rs. } 48000[1 + (8/200)]^2(1) \\
 &= \text{Rs. } 48000[1 + (1/25)]^2 \\
 &= \text{Rs. } 48000[(25 + 1)/25]^2 \\
 &= \text{Rs. } 48,000 \times (26/25) \times (26/25) \\
 &= \text{Rs. } 76.8 \times 26 \times 26 \\
 &= \text{Rs } 51,916.80
 \end{aligned}$$

Therefore, the compound interest = A – P

$$= \text{Rs } (51,916.80 - 48,000) = \text{Rs } 3,916.80$$

Ex. 5. If principal = Rs 1,00,000. rate of interest = 10% compounded half-yearly. Find

- (i) Interest for 6 months.
- (ii) Amount after 6 months.
- (iii) Interest for the next 6 months.
- (iv) Amount after one year.

Sol.

Given,

$$P = \text{Rs } 1,00,000$$

$$R = 10\%$$

$$(i) A = P[1 + (R/200)]^{2n}$$

Here, 2n is the number of half years.

Let us find the interest compounded half-yearly for 6 months, i.e., one half year.

$$\text{So, } A = \text{Rs. } 1,00,000 [1 + (10/200)]^1$$

$$= \text{Rs. } 1,00,000 [(20 + 1)/20]$$

$$= \text{Rs. } 1,00,000 \times 21/20$$

$$= \text{Rs. } 1,05,000$$

Compounded interest for 6 months = Rs. 1,05,000 – Rs. 1,00,000 = Rs. 5000

(ii) Amount after 6 months = Rs. 1,05,000

(iii) To find the interest for the next 6 months, we should consider the principal amount as Rs. 1,05,000.

Thus, $A = \text{Rs. } 1,05,000 [1 + (10/200)]^1$

= Rs. 1,05,000 × (21/20)

= Rs. 1,10,250

Compound interest for next 6 months = Rs. 1,10,250 – Rs. 1,05,000 = Rs. 5250

(iv) Amount after one year = Rs. 1,10,250

Let Us Sum Up

Compound interest occurs when the interest earned over time is added to the principal, so future interest calculations are based on the new total amount. For annual compounding, the formula to calculate the amount is $\text{Amount} = P \left(1 + \frac{R}{100}\right)^n$, where P is the principal, R is the annual interest rate, and n is the number of years. For half-yearly compounding, the formula adjusts to $\text{Amount} = P \left(1 + \frac{R/2}{100}\right)^{2n}$, and for quarterly compounding, it's $\text{Amount} = P \left(1 + \frac{R/4}{100}\right)^{4n}$. When dealing with fractional years, such as $3 \frac{2}{5}$ years, the calculation involves separately computing the amounts for whole and fractional years. For example, if Rs. 12,600 is invested at 10% per annum for 2 years, the compound interest calculated is Rs. 2,646. If the rate is unknown, like in finding the rate when Rs. 1,200 grows to Rs. 1,348.32 in 2 years, the rate can be determined by solving the equation for R, yielding 6%. Depreciation is similarly calculated, where a TV's value reducing by 5% per annum results in Rs. 18,004.875 after 3 years. For half-yearly compounding, such as with Rs. 48,000 at 8% per annum, the compound interest is Rs. 3,916.80 after a year, calculated using adjustments for semi-annual compounding.

Check your Progress

1. What is the formula to calculate the amount when interest is compounded quarterly?

a) Amount = $P \left(1 + \frac{R}{100}\right)^n$

b) Amount = $P \left(1 + \frac{R/2}{100}\right)^{2n}$

c) Amount = $P \left(1 + \frac{R/4}{100}\right)^{4n}$

d) Amount = $P \left(1 - \frac{R}{100}\right)^n$

2. If the principal amount is Rs. 10,000 and the annual interest rate is 8% compounded half-yearly, what is the amount after 1 year?

a) Rs. 10,800

b) Rs. 10,816

c) Rs. 10,832

d) Rs. 10,880

3. A sum of Rs. 5,000 becomes Rs. 5,800 in 2 years with interest compounded annually. What is the compound interest rate per annum?

a) 8%

b) 9%

c) 10%

d) 12%

4. For how many years is a principal of Rs. 2,000 invested if it grows to Rs. 2,656 at an annual compound interest rate of 8%?

a) 2 years

b) 3 years

c) 4 years

d) 5 years

5. A TV originally worth Rs. 30,000 depreciates by 6% per annum. What will be its value after 2 years?

- a) Rs. 26,796
- b) Rs. 25,488
- c) Rs. 24,000
- d) Rs. 22,512

3.5. SHAPES

Shapes are geometric figures that have defined boundaries, such as lines and curves, and can be described using mathematical properties. Here are some common shapes:

Circle

- Defined by a set of points equidistant from a fixed point called the centre.
- Properties: Radius, diameter, circumference, area.

Triangle

- A polygon with three sides and three angles.
- Properties: Side lengths, angles (acute, obtuse, right), perimeter, area.

Square

- A quadrilateral with four equal sides and four right angles.
- Properties: Side length, diagonal length, perimeter, area.

Rectangle

- A quadrilateral with opposite sides equal and four right angles.
- Properties: Length, width, diagonal length, perimeter, area.

Parallelogram

- A quadrilateral with opposite sides parallel and equal in length.
- Properties: Base, height, opposite angles, perimeter, area.

Rhombus

- A parallelogram with all sides equal in length.
- Properties: Side length, diagonal length, angles, perimeter, area.

Trapezoid

- A quadrilateral with one pair of parallel sides.
- Properties: Bases, height, non-parallel sides, perimeter, area.

Pentagon

- A polygon with five sides and five angles.
- Properties: Side lengths, interior angles, perimeter, area.

Hexagon

- A polygon with six sides and six angles.
- Properties: Side lengths, interior angles, perimeter, area.

Octagon

- A polygon with eight sides and eight angles.
- Properties: Side lengths, interior angles, perimeter, area.

These are just a few examples of basic shapes. There are many other shapes, including irregular polygons, curves like ellipses and hyperbolas, and three-dimensional shapes like cubes, cylinders, and spheres. Each shape has its own set of properties and formulas for calculating various measurements such as perimeter, area, and volume.

Let Us Sum Up

Shapes are geometric figures defined by boundaries such as lines and curves, and each has unique mathematical properties. A **circle** is characterized by points equidistant from a center, with key properties including radius and circumference. A **triangle** features three sides and angles, with properties like side lengths and area. A **square** has four equal sides and right angles, while a rectangle has opposite sides equal and four right angles. A **parallelogram** has opposite sides

parallel and equal, and a **rhombus** is a parallelogram with all sides equal. A **trapezoid** has one pair of parallel sides. **Pentagons, hexagons,** and **octagons** are **polygons** with five, six, and eight sides, respectively, each having specific side lengths and interior angles. Beyond these, there are irregular polygons and curves like ellipses, as well as three-dimensional shapes like cubes and spheres, each with its own set of properties and measurement formulas.

Check your Progress

1. Which of the following properties is specific to a circle?
 - a) Side length
 - b) Radius
 - c) Diagonal length
 - d) Base
2. What is a common property of both squares and rectangles?
 - a) All sides are equal
 - b) Four right angles
 - c) Opposite sides are parallel
 - d) The sum of interior angles is 360 degrees
3. In a parallelogram, which property is true?
 - a) All sides are equal in length
 - b) Opposite sides are parallel and equal
 - c) It has exactly two pairs of parallel sides
 - d) All angles are right angles
4. Which shape has exactly one pair of parallel sides?
 - a) Rhombus
 - b) Trapezoid
 - c) Square
 - d) Pentagon
5. Which of the following shapes is characterized by having six sides and six angles?
 - a) Pentagon

- b) Hexagon
- c) Octagon
- d) Triangle

3.6. DIVIDENDS

Dividends are a portion of a company's profits that are distributed to its shareholders as a return on their investment in the company's stock. Here are some key points about dividends:

Types of Dividends

1. Cash Dividends:

- Paid out in the form of cash to shareholders.
- Usually paid quarterly, but can be paid annually or semi-annually.

2. Stock Dividends:

- Paid out in the form of additional shares of stock rather than cash.
- Issued to shareholders at a predetermined ratio to their existing holdings.

3. Dividend Reinvestment Plans (DRIPs):

- Allow shareholders to automatically reinvest their cash dividends to purchase additional shares of stock.

Importance of Dividends

Income Generation

Provide shareholders with a regular source of income, especially for investors seeking income from their investments.

Signal of Financial Health

Companies that consistently pay dividends signal stability and confidence in their financial health and future prospects.

Total Return

Contribute significantly to the total return of an investment, along with capital appreciation.

Factors Affecting Dividends

Company Performance

Dividends are usually paid out of profits, so a company's financial performance directly impacts its ability to pay dividends.

Industry Trends

Industries with stable cash flows and mature businesses tend to pay higher dividends.

Economic Conditions

Economic downturns or recessionary periods may lead companies to reduce or suspend dividend payments.

Company Policies

Some companies have a consistent dividend policy, while others may prioritize reinvestment for growth.

Risks

1. Dividend Cuts:

Companies may cut or suspend dividend payments if they experience financial difficulties.

2. Market Volatility:

Changes in market conditions and investor sentiment can affect stock prices and dividend yields.

3. Inflation:

High inflation rates can erode the purchasing power of dividends over time if they do not keep pace with inflation.

Understanding dividends and their implications is important for investors in evaluating the potential returns and risks associated with investing in stocks.

Let Us Sum Up

Dividends are portions of a company's profits distributed to shareholders as a return on their investment. They can be in the form of cash dividends, which are paid out quarterly or at other intervals, stock dividends, where additional shares are issued instead of cash, or through Dividend Reinvestment Plans (DRIPs) that allow shareholders to reinvest dividends to buy more shares. Dividends are important as they provide a steady income, signal a company's financial health, and contribute to the total return of an investment. Factors affecting dividends include a company's performance, industry trends, economic conditions, and company policies. Risks associated with dividends include potential cuts if a company faces financial trouble, market volatility affecting stock prices and yields, and inflation diminishing the purchasing power of dividends. Understanding these aspects is crucial for assessing the potential returns and risks of stock investments.

Check your Progress

1. Which of the following describes a cash dividend?
 - a) Additional shares of stock distributed to shareholders
 - b) A portion of profits paid directly to shareholders in cash
 - c) Automatic reinvestment of dividends into additional shares
 - d) A form of dividend paid out annually only
2. What is a Dividend Reinvestment Plan (DRIP)?
 - a) A plan that allows shareholders to receive dividends in cash
 - b) A policy for companies to distribute dividends semi-annually
 - c) A plan that automatically reinvests cash dividends to purchase more shares
 - d) A scheme to pay dividends based on the company's financial health
3. What does a consistent dividend payment usually indicate about a company?
 - a) Financial instability
 - b) Lack of growth opportunities
 - c) Strong financial health and stability
 - d) Low investor confidence
4. Which factor is least likely to influence a company's ability to pay dividends?

- a) Company performance
- b) Industry trends
- c) Stock price volatility
- d) Economic conditions

5. What risk is associated with high inflation concerning dividends?

- a) Increased cash dividend payouts
- b) Erosion of the purchasing power of dividends
- c) Higher dividend yields
- d) Increased stock price appreciation

Unit Summary

This unit focuses on fundamental concepts of financial arithmetic, with applications in business, economics, and daily life. Here's an overview:

1. Percentages

- Definition: Percentages represent a ratio as a fraction of 100.
- Applications: Widely used in discounts, profit/loss calculations, tax computations, interest rates, and more.
- Key Concepts:
 - Converting fractions and decimals to percentages.
 - Calculating percentage increase/decrease.
 - Percentage of a quantity.

2. Profit and Loss

- Profit: The financial gain when selling price is greater than the cost price.
- Loss: Occurs when the cost price exceeds the selling price.
- Applications: Essential in business to calculate gains or losses in transactions.
- Formulas:
 - Profit = Selling Price - Cost Price
 - Profit Percentage = $(\text{Profit} / \text{Cost Price}) \times 100$
 - Loss = Cost Price - Selling Price
 - Loss Percentage = $(\text{Loss} / \text{Cost Price}) \times 100$

3. Discount

- Definition: A reduction in the original price of goods/services.
- Applications: Common in retail and marketing strategies to attract customers.
- Formula:
 - Discount = Marked Price - Selling Price
 - Discount Percentage = (Discount / Marked Price) × 100

4. Simple Interest

- Definition: Interest calculated only on the original principal over a period of time.
- Formula:
 - $SI = (P \times R \times T) / 100$
 - Where, P = Principal, R = Rate of Interest, T = Time
- Applications: Used in short-term loans, investments, and savings accounts.

5. Compound Interest

- Definition: Interest calculated on the initial principal, including all accumulated interest from previous periods.
- Formula:
 - $CI = P (1 + R/100)^T - P$
- Applications: Used in long-term savings, investments, and loans.

6. Partnerships

- Definition: A business arrangement where two or more individuals share profits and losses based on their investment or agreed terms.
- Formula:
 - Ratio of division = (Investment of partner A / Investment of partner B)
- Applications: Common in joint ventures and business collaborations.

7. Shares

- Definition: Units of ownership in a company.
- Application: Buying shares gives the owner a claim on part of the company's profits and assets.
- Types:
 - Common Shares: Provide voting rights and dividends.
 - Preferred Shares: Provide a fixed dividend without voting rights.

8. Dividends

- Definition: A portion of a company's profits distributed to its shareholders.
- Applications: Regular income for shareholders and an indicator of a company's profitability.
- Formula:
 - Dividend Yield = (Dividend per Share / Market Price of Share) × 100

Glossary

1. Principal: The original amount of money invested or loaned.
2. Interest Rate: The percentage charged on a loan or paid on an investment.
3. Cost Price (CP): The price at which an item is purchased.
4. Selling Price (SP): The price at which an item is sold.
5. Marked Price (MP): The original listed price before any discount.
6. Dividend: The portion of a company's earnings distributed to shareholders.
7. Profit Percentage: The percentage gain over the cost price.
8. Loss Percentage: The percentage loss relative to the cost price.
9. Shareholder: An individual or entity owning shares in a company.
10. Yield: The earnings generated and realized on an investment over a specific period.

UNIT-IV

DATA INTERPRETATION

The term "average" typically refers to a central or typical value within a dataset and is commonly used in statistics and mathematics.

4.1. AVERAGE

An average is a single value that represents the typical value or central tendency of a set of numbers. It is calculated by summing up all the numbers in the dataset and dividing by the total count of numbers. Averages are used to summarize and analyse data, providing insights into the general characteristics of a dataset.

4.1.1. Types of Averages:

1. Mean: The most common type of average, calculated by dividing the sum of all values by the total number of values.
2. Median: The middle value in a dataset when the values are arranged in ascending or descending order. If there is an even number of values, the median is the average of the two middle values.
3. Mode: The value that appears most frequently in a dataset.

Solved Problems

Mean

Ex. 1. Calculate the mean from the data showing marks of students in a class in a test: 40, 50, 55, 78, 58.

Sol.

Given marks:

40, 50, 55, 78, 58

Here, the number of data values = 5

We know that:

Mean = $\frac{\text{Sum of data values}}{\text{Total number of data values}}$

$$= (40 + 50 + 55 + 78 + 58)/5$$

$$= 281/5$$

$$= 56.2$$

Therefore, the mean for the given data is 56.2.

Ex. 2. A total of 25 patients admitted to a hospital are tested for levels of blood sugar, (mg/dl) and the results obtained were as follows:

87, 71, 83, 67, 85, 77, 69, 76, 65, 85, 85, 54, 70, 68, 80, 73, 78, 68, 85, 73, 81, 78, 81, 77, 75

Find the mean (mg/dl) of the above data.

Sol.

$$\begin{aligned} \text{Sum of data values} &= 87 + 71 + 83 + 67 + 85 + 77 + 69 + 76 + 65 + 85 + 85 \\ &+ 54 + 70 + 68 + 80 + 73 + 78 + 68 + 85 + 73 + 81 + 78 + 81 + 77 + 75 \end{aligned}$$

$$= 1891$$

$$\text{Mean} = 1891/25$$

$$= 75.64$$

Ex. 3. Find the mean of the first 10 odd integers.

Sol.

First 10 odd integers: 1, 3, 5, 7, 9, 11, 13, 15, 17, 19

Mean = Sum of the first 10 odd integers/Number of such integers

$$= (1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19)/10$$

$$= 100/10$$

$$= 10$$

Median

Ex. 1. What is the median of the following data set?

32, 6, 21, 10, 8, 11, 12, 36, 17, 16, 15, 18, 40, 24, 21, 23, 24, 24, 29, 16, 32, 31, 10, 30, 35, 32, 18, 39, 12, 20

Sol.

The ascending order of the given data set is:

6, 8, 10, 10, 11, 12, 12, 15, 16, 16, 17, 18, 18, 20, 21, 21, 23, 24, 24, 24, 29, 30, 31, 32, 32, 32, 35, 36, 39, 40

Number of values in the data set = $n = 30$

$$n/2 = 30/2 = 15$$

15th data value = 21

$$(n/2) + 1 = 16$$

16th data value = 21

$$\text{Median} = [(n/2)\text{th observation} + \{(n/2)+1\}\text{th observation}]/2$$

$$= (15\text{th data value} + 16\text{th data value})/2$$

$$= (21 + 21)/2$$

$$= 21$$

Ex. 2. The runs scored by 11 players in the cricket match are as follows:

7, 16, 121, 51, 101, 81, 1, 16, 9, 11, 16

Find the median of the data.

Sol.

Given data: 7, 16, 121, 51, 101, 81, 1, 16, 9, 11, 16.

Now, arrange the data in ascending order, we get

1, 7, 9, 11, 16, 16, 16, 51, 81, 101, 121.

Here, the number of observations is 11, which is odd.

Thus, median = 6th term

Hence, the median of the given data is 16.

Ex. 3. What is the median of 4, 2, 7, 3, 10, 9, 13?

Sol.

Given data: 4, 2, 7, 3, 10, 9, 13

First, arrange the given data in ascending order.

i.e., 2, 3, 4, 7, 9, 10, 13

So, the number of observations is 7. ($n = 7$)

Since the number of observations is odd, the median can be calculated as follows:

Median = $[(n + 1)/2]$ th term

Median = $[(7 + 1)/2]$ th term

Median = $[(8)/2]$ th term

Median = 4th term = 7

Hence, the median of 4, 2, 7, 3, 10, 9, 13 is 7.

Mode

Ex. 1. The following are the marks scored by 20 students in the class. Find the mode
90, 70, 50, 30, 40, 86, 65, 73, 68, 90, 90, 10, 73, 25, 35, 88, 67, 80, 74, 46

Sol.

Since the marks 90 occurs the maximum number of times, three times compared with the other numbers, mode is 90.

Ex. 2. Compute mode value for the following observations.

2, 7, 10, 12, 10, 19, 2, 11, 3, 12

Sol. Here, the observations 10 and 12 occurs twice in the data set, the modes are 10 and 12.

Importance of Averages:

- Averages provide a summary measure of central tendency, allowing for easy comparison and understanding of data.
- They help in making decisions, predictions, and drawing conclusions based on data analysis.
- Averages are widely used in various fields such as finance, economics, science, and social sciences for analysing trends, forecasting, and decision-making. Understanding averages is fundamental in statistical analysis and data interpretation, enabling researchers, analysts, and decision-makers to gain insights from datasets and draw meaningful conclusions.

Let Us Sum Up

The term "**average**" represents a central value within a dataset and is a key concept in statistics and mathematics. There are three main types of averages: the **mean**, calculated by dividing the sum of all values by their count; the **median**, the middle value in an ordered dataset; and the **mode**, the most frequently occurring value. For example, to find the mean, you sum the data values and divide by the total number. The median is found by arranging data in order and identifying the middle value, while the mode is the value that appears most often. Averages help summarize data, compare datasets, and make informed decisions in fields like finance, economics, and social sciences. They are crucial for analyzing trends and drawing conclusions from data.

Check your Progress

1. Which type of average is calculated by summing all the values in a dataset and dividing by the number of values?
 - a) Median
 - b) Mode
 - c) Mean
 - d) Range
2. If a dataset has the following values: 5, 7, 9, 11, and 15, what is the median?
 - a) 7

- b) 9
 - c) 11
 - d) 10
3. In the dataset 3, 7, 7, 8, 9, 9, 9, which value is the mode?
- a) 3
 - b) 7
 - c) 8
 - d) 9
4. What is the mean of the numbers 2, 4, 6, 8, and 10?
- a) 4
 - b) 5
 - c) 6
 - d) 8
5. Which type of average is best used when you want to find the middle value in an ordered dataset?
- a) Mean
 - b) Median
 - c) Mode
 - d) Range

4.2. MIXTURE AND ALLEGATION

Mixture refers to the mix that is derived as a result of mixing two or more items or substances in a certain ratio or proportion. Usually, mixtures can be made by mixing solids, liquids or gases with other solids, liquids or gases. However, a mixture can be also derived by mixing any combination of solids, liquids and gases.

Allegation is a rule that helps us solve problems related to mixtures. Allegation rule helps in finding out the ratio in which two items or ingredients, having a certain cost must be mixed to obtain a final mixture having ingredients in a known ratio.

Allegation Rule: Allegation problems involve the application of the weighted average concept of average. You must be already aware that if we have 1, 2, 3,, k sets, each containing $n_1, n_2, n_3, \dots, n_k$ elements, and the average of set k is represented by A_k then the weighted average of all the sets together is calculated as

$$Aw = \frac{n_1 A_1 + n_2 A_2 + \dots + n_k A_k}{n_1 + n_2 + \dots + n_k}$$

Generally, many questions mention that two ingredients with different prices are mixed together to form a mixture. In such cases, the following formula is considered,

$$\frac{\text{Quantity of cheaper ingredient}}{\text{Quantity of dearer ingredient}} = \frac{\text{Cost of dearer ingredient} - \text{Mean Price}}{\text{Mean Price} - \text{Cost price of a cheaper ingredient}}$$

The price per unit of the final mixture is called the mean price or weighted price.

Graphical Solution of Allegation

We can also solve the allegation problems by using a graphical approach instead of doing it with the help of equations. The graphical method will look like a cross which is why it is also known as the cross method.

Types of Questions asked in Mixture and Allegation

Various types of questions are asked from the mixture and allegation. Some of them are as follows.

(a) Mixture of Two Things: In these types of questions we make a mixture of two things.

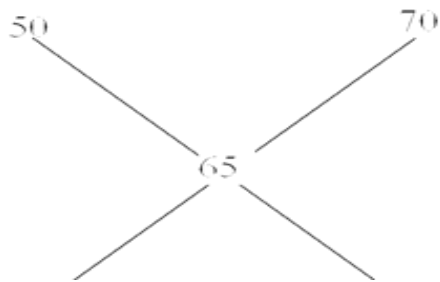
(b) Two mixtures are mixed to form a new mixture: In these types of questions, two or more mixtures are mixed together and form a new mixture, on which the questions will be asked.

(c) Selling of Mixtures: In these types of questions we make a mixture of two things and sell the resultant mixture.

Solved Problems

Ex. 1. In what ratio must a shopkeeper mix two types of rice worth Rs. 50 kg and Rs. 70 kg, so that the average cost of the mixture is Rs. 65 kg?

Sol. By Rule of allegation,



$$\frac{\text{Cheaper quantity}}{\text{Dearer quantity}} = \frac{(70-65)}{(65-50)} = \frac{5}{15} = 1:3 \quad \text{OR}$$

Hence the answer is 5:15 or 1:3.

Ex. 2. The ratio of milk and water in a solution is 20: 7 and after adding 5 liters of water in it the ratio of milk and water becomes 5: 3, then find the final amount of water in the final solution.

Sol. Let the initial amount of milk be $20x$ and of water $7x$.

$$\text{Ratio of milk and water after adding 5 litres} = 20x / (7x + 5) = 5/3$$

$$\Rightarrow 60x = 35x + 25$$

$$\Rightarrow 25x = 25$$

$$\Rightarrow x = 1.$$

$$\therefore \text{Final amount of water in solution} = 7x + 5 = 7 + 5 = 12 \text{ litres. Smart approach}$$

$$\text{Initial ratio of milk and water} = 20/7 \text{ — (1)}$$

$$\text{Final ratio of milk and water} = 5/3 \text{ — (2)}$$

$$\text{Multiplying equation 2 with 4 (to make amount of milk equal), we get Final ratio of milk and water} = 20/12 \text{ — (3)}$$

\therefore Amount of water in final solution = 12 litter.

Ex. 3. Two vessels of equal capacity contain juice and water in the ratio of 7: 2 and 11: 7 respectively. The mixture of both vessels is mixed and transferred into a bigger vessel. What is the ratio of juice and water in the new mixture?

Sol. The ratio of juice and water in the first vessel = 7: 2 — (1)

Total capacity of first vessel = $7 + 2 = 9$ units

The ratio of juice and water in the second vessel = 11: 7 — (2)

Total capacity of second vessel = $11 + 7 = 18$ units

We will have to equal the total capacity of both vessels, so multiply by 2 in equation (1).

The ratio of juice and water in the first vessel = 14: 4 — (3)

The ratio of juice and water in the second vessel = 11: 7 — (4)

Ratio of juice and water in bigger vessel = $(14 + 11): (4 + 7) = 25: 11$

Let Us Sum Up

A mixture is created by combining two or more substances, which can be solids, liquids, or gases, in various proportions. Allegation is a method used to determine the ratio in which different ingredients should be mixed to achieve a desired cost or composition. It involves using weighted averages, where the average price or value of the mixture is calculated based on the proportions and costs of the ingredients. The formula for this is based on the difference between the cost of the more expensive and less expensive ingredients and the desired mean price. Problems involving mixtures can include mixing two ingredients, combining different mixtures, or selling mixtures. Graphical solutions, such as the cross method, can also be used for these calculations. For example, if two types of rice are mixed to achieve a specific average cost, the ratio can be found using the allegation rule. Similarly, when mixing juices or solutions with different ratios, the final mixture's composition can be determined by adjusting the proportions and capacities of the original mixtures.

Check your Progress

1. What does the allegation rule help determine in mixture problems?
 - a) The total volume of the mixture
 - b) The ratio of ingredients required to achieve a desired mixture composition
 - c) The temperature of the mixture
 - d) The density of the mixture

2. If a shopkeeper mixes two types of rice costing Rs. 40 per kg and Rs. 60 per kg to get a mixture costing Rs. 50 per kg, what is the ratio of the two types of rice?
 - a) 1:1
 - b) 2:1
 - c) 1:2
 - d) 3:2

3. In a mixture of juice and water, if the initial ratio of juice to water is 3:2 and after adding 5 liters of water the ratio becomes 2:1, what was the initial amount of water in the mixture?
 - a) 5 liters
 - b) 10 liters
 - c) 15 liters
 - d) 20 liters

4. Two vessels contain mixtures of juice and water with ratios 7:2 and 11:7 respectively. If the mixtures are combined, what is the ratio of juice to water in the new mixture?
 - a) 14:9
 - b) 25:11
 - c) 18:11
 - d) 7:5

5. When using the graphical method of allegation, which shape is commonly used to represent the mixing of different ingredients?

- a) Square
- b) Circle
- c) Cross
- d) Triangle

4.3. BAR DIAGRAM

Bar charts are graphical representations of data that use bars to compare different categories or groups. They are commonly used to display and compare the frequency, count, or values of discrete categories. Here's an overview of bar charts:

Components of a Bar Chart:

Bars

- Rectangular bars represent the categories or groups being compared.
- The length or height of each bar corresponds to the value it represents.

Axes

- The horizontal axis (x-axis) typically represents the categories or groups.
- The vertical axis (y-axis) represents the scale of measurement or frequency.

Labels

- Each axis is labelled to indicate the categories or the scale of measurement.
- Labels may also be included on or near the bars to provide additional information.

Title

- A title is provided to describe the overall content or purpose of the bar chart.

4.3.1. Types of Bar Charts:

1. Vertical Bar Chart

- Bars are drawn vertically, with the height of each bar representing the value of the category.

- Used when comparing categories or groups along the horizontal axis.

2. Horizontal Bar Chart

- Bars are drawn horizontally, with the length of each bar representing the value of the category.
- Used when comparing categories or groups along the vertical axis.

Example

Consider a bar chart representing the sales revenue of a company's products in a particular month:

- Categories (Products): A, B, C, D
- Sales Revenue (in \$): 5000, 7000, 4000, 6000

In this example:

- Vertical bars represent each product (A, B, C, D).
- The height of each bar corresponds to the sales revenue of the respective product.
- The horizontal axis represents the product categories, and the vertical axis represents the sales revenue scale.
- The title "Sales Revenue by Product" provides an overview of the chart's content.

Importance of Bar Charts

1. Visual Comparison

Bar charts provide a visual means to compare categories or groups easily.

2. Data Representation

They effectively represent discrete data, making them suitable for categorical data analysis.

3. Clarity and Interpretation

Bar charts are simple and easy to understand, making them accessible to a wide audience.

4. Identifying Trends and Patterns

They help in identifying trends, patterns, and outliers in the data.

5. Communication

Bar charts are widely used in presentations, reports, and publications to communicate findings and insights effectively.

Bar charts are versatile and widely used in various fields, including business, economics, finance, and social sciences, to visualize and analyse categorical data.

ADVANTAGES AND DISADVANTAGES OF BAR CHART

Advantages:

- Bar graph summarises the large set of data in simple visual form.
- It displays each category of data in the frequency distribution.
- It clarifies the trend of data better than the table.
- It helps in estimating the key values at a glance.

Disadvantages:

- Sometimes, the bar graph fails to reveal the patterns, cause, effects, etc.
- It can be easily manipulated to yield fake information.\

Let Us Sum Up

Bar charts are graphical tools used to compare different categories or groups by representing data with rectangular bars. The length or height of each bar corresponds to the value of the category it represents. Key components include the horizontal axis (x-axis) for categories, the vertical axis (y-axis) for measurement scale, labels for clarity, and a title for context. Bar charts come in two types: vertical, where bars extend upward to show values, and horizontal, where bars extend sideways. They are effective for visual comparison, data representation, and identifying trends or patterns. While bar charts offer a clear and accessible means of displaying categorical data, they can sometimes fail to reveal detailed patterns and may be susceptible to manipulation. Overall, they are widely used in various fields to simplify complex data for analysis and presentation.

Check your Progress

1. In a vertical bar chart, which axis typically represents the categories or groups?
 - a) Vertical axis
 - b) Horizontal axis
 - c) Diagonal axis
 - d) None of the above

2. What does the length or height of a bar in a bar chart represent?
 - a) The color of the bar
 - b) The category of the data
 - c) The value or frequency of the category
 - d) The title of the chart

3. Which type of bar chart would be more appropriate if you need to compare data categories with longer names?
 - a) Vertical Bar Chart
 - b) Horizontal Bar Chart
 - c) Line Chart
 - d) Pie Chart

4. What is one disadvantage of using a bar chart?
 - a) It provides a visual means to compare categories
 - b) It can clarify trends and patterns
 - c) It may fail to reveal detailed patterns and can be manipulated to misrepresent information
 - d) It helps in estimating key values at a glance

5. In the bar chart example showing sales revenue of products A, B, C, and D, what does the title "Sales Revenue by Product" indicate?
 - a) The types of products
 - b) The purpose of the chart
 - c) The color scheme used
 - d) The exact numerical values of the sales

4.4. PIE DIAGRAM

Pie charts are circular graphical representations of data in which the circle is divided into sectors or slices to illustrate numerical proportion. Each slice represents a proportionate part of the whole dataset. Here's an overview of pie charts:

Components of a Pie Chart:

1. Circle

The entire circle represents the total or 100% of the data.

2. Slices

- Each slice of the pie represents a category or group.
- The size of each slice is proportional to the quantity or percentage it represents.

3. Labels

- Labels are typically placed either inside or outside each slice to identify the corresponding category.
- They may include the category name and/or the percentage or value it represents.

4. Legend (Optional)

- A legend may be included to provide additional information about the categories represented in the pie chart.

5. Title

- A title is provided to describe the overall content or purpose of the pie chart.

Example:

Consider a pie chart representing the distribution of expenses in a household budget:

Categories (Expenses): Housing, Transportation, Food, Utilities, Entertainment

Proportions: 30%, 20%, 25%, 15%, 10%

In this example:

- Each slice of the pie represents a category of expenses.

- The size of each slice is proportional to the percentage of the total budget spent on that category.
- Labels inside or outside each slice indicate the category name and its percentage of the total.

4.4.1. Types of Pie Charts:

1. Standard Pie Chart

A standard pie chart represents the data with slices arranged in a circular manner.

2. Exploded Pie Chart

In an exploded pie chart, one or more slices are separated or "exploded" from the center of the pie for emphasis.

3. Donut Chart

A donut chart is similar to a pie chart but with a hole in the center. It can display multiple data series.

Importance of Pie Charts

1. Visual Representation

Pie charts provide a visually appealing way to represent proportions and percentages.

2. Comparison of Parts to the Whole

They help in comparing the relative sizes of different categories to the total.

3. Easy Interpretation

Pie charts are easy to understand and interpret, even for individuals with little statistical knowledge.

4. Highlighting Key Categories

They allow for the highlighting of key categories or segments of the data.

5. Effective Communication

Pie charts are commonly used in presentations, reports, and publications to communicate key findings and insights. Pie charts are effective tools for visualizing and analysing data distributions, particularly when dealing with categorical data and relative proportions. However, they are best suited for representing simple datasets with a small number of categories.

Let Us Sum Up

Pie charts are circular graphs that illustrate data proportions by dividing the circle into slices, with each slice representing a category or group. The size of each slice is proportional to its percentage of the whole dataset, making it easy to visualize and compare different categories. Components of a pie chart include the circle itself, slices, labels for identifying categories, and sometimes a legend. The chart's title provides context for the data being represented. Types of pie charts include standard, exploded (with separated slices for emphasis), and donut charts (with a central hole). Pie charts are valuable for visually representing proportions, comparing parts to a whole, and highlighting key categories. They are straightforward to interpret, making them useful in presentations and reports. However, they are best suited for simple datasets with a limited number of categories.

Check your Progress

1. What does each slice in a pie chart represent?
 - a) A different category or group
 - b) A unit of measurement
 - c) The total data set
 - d) The chart title
2. Which type of pie chart features one or more slices separated from the center for emphasis?
 - a) Standard Pie Chart
 - b) Donut Chart
 - c) Exploded Pie Chart
 - d) 3D Pie Chart

3. In a pie chart showing household expenses, if the slice for 'Housing' represents 30% of the total budget, what does this mean?

- a) Housing is the least expensive category.
- b) Housing makes up one-third of the total expenses.
- c) Housing is the most expensive category.
- d) Housing represents the smallest proportion of the budget.

4. What is a primary advantage of using pie charts?

- a) They provide a detailed view of each data point.
- b) They are ideal for displaying data trends over time.
- c) They offer a visual representation of proportions and percentages.
- d) They can represent large datasets with many categories.

5. Which type of pie chart has a hole in the center and can display multiple data series?

2. Standard Pie Chart
3. Exploded Pie Chart
4. 3D Pie Chart
5. Donut Chart

4.4. VENN DIAGRAM

Venn diagrams are graphical representations used to illustrate the relationships between different sets or groups of items. They consist of overlapping circles, each representing a set, and the overlapping areas represent the intersections or common elements between the sets. Here's an overview of Venn diagrams:

Components of a Venn Diagram

1. Circles

- Each circle represents a set or group of items.
- The size of the circles may vary depending on the number of elements in each set.

2. Overlap

The overlapping areas between circles represent the intersection or common elements shared by the sets.

3. Labels

Labels are used to identify the sets and may be placed inside or outside the circles.

4. Regions

Regions within and outside the circles represent specific combinations of elements or groups.

Example

Consider a Venn diagram representing the relationship between different types of fruits:

Set A: Apples

Set B: Bananas

Overlapping Area: Fruits that are both apples and bananas

In this example:

Circle A represents the set of apples.

Circle B represents the set of bananas.

The overlapping area represents fruits that are both apples and bananas.

4.5.1. Types of Venn Diagrams

1. Two-set Venn Diagram

Consists of two circles representing two sets and their intersection.

2. Three-set Venn Diagram

Consists of three circles representing three sets and their intersections.

3. Multi-set Venn Diagram

Consists of more than three circles representing multiple sets and their intersections.

Uses of Venn Diagrams

1. Set Operations

Venn diagrams are used to visualize set operations such as union, intersection, and complement.

2. Logic and Probability

They are used in logic and probability to represent relationships between events and outcomes.

3. Data Analysis

Venn diagrams are used in data analysis to compare and contrast different groups or categories of data.

4. Problem-solving

They are used in problem-solving to organize and visualize information, particularly in logic puzzles and word problems.

Importance of Venn Diagrams:

1. Visual Representation

Venn diagrams provide a visual representation of relationships between sets, making complex concepts easier to understand.

2. Clarity and Organization

They help in organizing and clarifying information, particularly when dealing with multiple sets or categories.

3. Analysis and Comparison

Venn diagrams facilitate analysis and comparison of different groups or categories of items.

4. Problem-solving Tool

They are effective problem-solving tools, particularly in fields such as mathematics, logic, and data analysis.

Venn diagrams are versatile tools that are widely used in various fields, including mathematics, logic, statistics, and data analysis, to visualize and analyse relationships between sets or groups of items.

Let Us Sum Up

Venn diagrams are visual tools used to illustrate relationships between different sets or groups. They consist of overlapping circles, where each circle represents a set and the overlapping areas show the intersections or common elements among the sets. Components of a Venn diagram include circles, which represent sets, overlapping regions indicating shared elements, and labels for identification. There are different types of Venn diagrams: two-set, three-set, and multi-set, depending on the number of circles involved. Venn diagrams are used for set operations like union and intersection, as well as in logic, probability, and data analysis. They help in visualizing complex relationships, organizing information, and solving problems by providing clear, organized comparisons of different groups.

Check your Progress

1. What does the overlapping area between circles in a Venn diagram represent?
 - a) The union of all sets
 - b) The intersection or common elements between the sets
 - c) The difference between the sets
 - d) The complement of the sets
2. Which type of Venn diagram involves three circles representing three sets and their intersections?
 - a) Two-set Venn Diagram
 - b) Three-set Venn Diagram
 - c) Multi-set Venn Diagram
 - d) Single-set Venn Diagram
3. In a Venn diagram, if Set A represents apples and Set B represents bananas, what does the overlapping area indicate?
 - a) The total number of fruits
 - b) Fruits that are both apples and bananas

- c) Fruits that are only bananas
 - d) Fruits that are only apples
4. Which of the following is NOT a typical use of Venn diagrams?
- a) Visualizing set operations like union and intersection
 - b) Analyzing and comparing different groups of data
 - c) Calculating financial ratios
 - d) Representing relationships between events and outcomes
5. What is the primary benefit of using Venn diagrams in problem-solving and data analysis?
- a) They provide numerical data analysis
 - b) They offer a visual representation that clarifies relationships between sets
 - c) They calculate probabilities directly
 - d) They are used to perform statistical regressions

Unit Summary

Data Interpretation

Averages

- Definition: Averages provide a measure of central tendency in a dataset. Common types include mean, median, and mode.
- Applications: Used in various fields such as finance, education, and science to summarize data.

Mixtures and Allegations

- Definition: Mixtures refer to combining two or more substances. The allegation method helps solve problems related to mixtures of different ratios and prices.
- Applications: Often used in business and chemistry to calculate cost or concentration.

Bar Charts

- Definition: A bar chart is a graphical representation of data using bars of different heights or lengths to show the values of different categories.
- Applications: Useful for comparing quantities across categories, such as sales by region or survey responses.

Pie Charts

- Definition: A pie chart is a circular statistical graphic that is divided into slices to illustrate numerical proportions.
- Applications: Commonly used to show percentage shares of a whole, such as market share or budget distribution.

Venn Diagrams

- Definition: A Venn diagram is a diagram that shows all possible logical relations between a finite collection of different sets.
- Applications: Useful for visualizing relationships and overlaps among different groups, such as survey data or sets in mathematics.

Glossary

1. Mean: The average of a set of numbers, calculated by dividing the sum of the values by the number of values.
2. Median: The middle value in a dataset when arranged in ascending or descending order.
3. Mode: The value that appears most frequently in a dataset.
4. Ratio: A relationship between two numbers indicating how many times the first number contains the second.
5. Proportion: A part or share of a whole, often expressed as a percentage.
6. Dataset: A collection of related data points or values, typically organized in a table or spreadsheet.
7. Graphical Representation: A visual way to present data, such as charts or graphs, to make it easier to understand and interpret.

8. Category: A class or division of items or data that share similar characteristics.
9. Set: A collection of distinct objects, considered as an object in its own right, often used in mathematics.
10. Overlap: The part of a Venn diagram where two or more sets share common elements.

UNIT – V

GEOMETRY AND MENSURATION

Geometry and mensuration (the study of measurements, shapes, and their properties) have significant applications in various industries. Here's how they are utilized:

5.1. GEOMETRY

1. Architecture and Construction

Geometry is crucial in designing and constructing buildings, bridges, and infrastructure. Architects use geometric principles to create blueprints, determine dimensions, and ensure structural stability.

2. Manufacturing

Geometry is applied in manufacturing processes such as machining, casting, and modelling. It helps in designing machine components, optimizing tool paths, and ensuring accurate production of parts.

3. Aerospace Industry

Geometry is used in aircraft design, aerodynamics, and space exploration. Engineers rely on geometric calculations to model air foils, analyse fluid dynamics, and design spacecraft components.

4. Automotive Industry

Geometry plays a vital role in automotive design, manufacturing, and testing. It is used to design vehicle components, optimize vehicle performance, and simulate crash tests.

5. Robotics and Automation

Geometry is essential in robotics for motion planning, path optimization, and manipulation tasks. It helps in determining robot trajectories, workspace analysis, and obstacle avoidance.

Let Us Sum Up

Geometry plays a pivotal role across various industries, influencing design and functionality. In architecture and construction, geometric principles are used to create blueprints, determine dimensions, and ensure structural integrity for buildings and infrastructure. In manufacturing, geometry aids in designing machine components, optimizing tool paths, and achieving precision in production. The aerospace industry relies on geometry for aircraft design, aerodynamics, and spacecraft components, utilizing geometric calculations for modeling and analysis. Similarly, the automotive industry applies geometry in designing vehicle parts, optimizing performance, and conducting crash simulations. In robotics and automation, geometry is crucial for motion planning, path optimization, and obstacle avoidance, helping to determine robot trajectories and workspace analysis. Overall, geometry is integral to designing, manufacturing, and optimizing various technological and structural systems.

Check your Progress

1. In which of the following industries is geometry used to design blueprints and ensure structural stability of buildings?
 - a) Aerospace Industry
 - b) Robotics and Automation
 - c) Architecture and Construction
 - d) Automotive Industry
2. Which industry utilizes geometric principles for optimizing tool paths and ensuring precision in part production?
 - e) Aerospace Industry
 - f) Manufacturing
 - g) Automotive Industry
 - h) Robotics and Automation

3. Geometry is crucial in analyzing fluid dynamics and designing spacecraft components in which industry?

- a) Manufacturing
- b) Automotive Industry
- c) Aerospace Industry
- d) Architecture and Construction

4. How is geometry applied in the automotive industry?

- a) To create blueprints for buildings
- b) To model air foils and spacecraft components
- c) To design vehicle components, optimize performance, and simulate crash tests
- d) To plan robot trajectories and avoid obstacles

5. In robotics and automation, what role does geometry play?

- a) Designing vehicle components
- b) Creating blueprints for buildings
- c) Motion planning, path optimization, and workspace analysis
- d) Analyzing fluid dynamics

5.2. MENSURATION

1. Construction and Land Surveying

Mensuration is used in construction projects and land surveying to measure distances, areas, and volumes of land parcels. Surveyors rely on mensuration techniques to establish property boundaries and create topographic maps.

2. Mining and Resource Extraction

Mensuration is applied in mining operations to calculate the volume of ore deposits, estimate resource reserves, and plan excavation activities. It helps in optimizing mining processes and maximizing resource extraction efficiency.

3. Agriculture and Farming

Mensuration is used in agriculture for field measurement, crop yield estimation, and irrigation planning. Farmers use mensuration techniques to determine planting densities, assess crop health, and optimize land use.

4. Logistics and Supply Chain Management

Mensuration plays a role in logistics and supply chain management for warehouse optimization, cargo loading, and transportation planning. It helps in calculating storage capacities, pallet configurations, and shipping volumes.

5. Environmental Monitoring

Mensuration is utilized in environmental monitoring and conservation efforts to measure land, water, and air quality parameters. It helps in assessing environmental impacts, tracking changes over time, and implementing remediation measures.

In summary, geometry and mensuration have diverse applications across industries, ranging from design and manufacturing to resource management and environmental monitoring. Their principles and techniques are essential for solving real-world problems and optimizing processes in various fields.

Solved problems

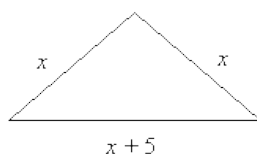
Ex. 1. A triangle has a perimeter of 50. If 2 of its sides are equal and the third side is 5 more than the equal sides, what is the length of the third side?

Sol.

Step 1: Assign variables:

Let x = length of the equal sides

Sketch the figure



Step 2: Write out the formula for perimeter of triangle.

P = sum of the three sides

Step 3: Plug in the values from the question and from the sketch.

$$50 = x + x + x + 5$$

Combine like terms

$$50 = 3x + 5$$

Isolate variable x

$$3x = 50 - 5$$

$$3x = 45$$

$$x = 15$$

Be careful! The question requires the length of the third side.

$$\text{The length of third side} = 15 + 5 = 20$$

Answer: The length of third side is 20.

Ex. 2. A circle has a radius of 21 cm. Find its circumference and area. (Use $\pi = 22/7$)

Sol. We know,

$$\text{Circumference of circle} = 2\pi r = 2 \times (22/7) \times 21 = 2 \times 22 \times 3 = 132 \text{ cm}$$

$$\text{Area of circle} = \pi r^2 = (22/7) \times 21^2 = 22/7 \times 21 \times 21 = 22 \times 3 \times 21$$

$$\text{Area of circle with radius, 21cm} = 1386 \text{ cm}^2$$

Ex. 3. If one side of a square is 4 cm, then what will be its area and perimeter?

Sol. Given,

$$\text{Length of side of square} = 4 \text{ cm}$$

$$\text{Area} = \text{side}^2 = 4^2 = 4 \times 4 = 16 \text{ cm}^2$$

$$\text{Perimeter of square} = \text{sum of all its sides}$$

Since, all the sides of the square are equal, therefore;

$$\text{Perimeter} = 4+4+4+4 = 16 \text{ cm}$$

Ex. 4. The height, length and width of a cuboidal box are 20 cm, 15 cm and 10 cm, respectively. Find its area.

Sol. Total surface area = $2(20 \times 15 + 20 \times 10 + 10 \times 15)$

$$\text{TSA} = 2(300 + 200 + 150) = 1300 \text{ cm}^2$$

Ex. 5. Find the height of a cuboid whose volume is 275 cm^3 and base area is 25 cm^2 .

Sol. Volume of cuboid = $l \times b \times h$

$$\text{Base area} = l \times b = 25 \text{ cm}^2$$

Hence,

$$275 = 25 \times h$$

$$h = 275/25 = 11 \text{ cm}$$

Let Us Sum Up

Mensuration plays a crucial role across various fields by providing essential techniques for measuring distances, areas, and volumes. In construction and land surveying, it helps determine property boundaries and create topographic maps. In mining, mensuration is used to calculate ore volumes and optimize resource extraction. Agriculture benefits from mensuration through field measurements, crop yield estimation, and irrigation planning. It also aids in logistics for warehouse optimization and cargo planning, as well as in environmental monitoring for assessing land, water, and air quality. Solved problems illustrate practical applications, such as finding the length of a triangle's side based on its perimeter, calculating a circle's circumference and area, determining a square's area and perimeter, and solving for dimensions of a cuboid. These examples show how mensuration techniques are applied to solve real-world problems and enhance efficiency in diverse sectors.

Check your Progress

1. In construction and land surveying, mensuration techniques are primarily used for:
 - a) Estimating building material costs
 - b) Measuring distances, areas, and volumes of land parcels
 - c) Designing structural blueprints
 - d) Conducting environmental impact assessments

2. In the mining industry, mensuration helps in:
 - a) Designing mining equipment
 - b) Calculating ore deposit volumes and planning excavation

- c) Evaluating the environmental impact of mining
 - d) Training staff for mining operations
3. If the length of one side of a square is 4 cm, what is its area?
- a) 8 cm²
 - b) 12 cm²
 - c) 16 cm²
 - d) 20 cm²
4. The total surface area (TSA) of a cuboidal box with height 20 cm, length 15 cm, and width 10 cm is:
- a) 1000 cm²
 - b) 1200 cm²
 - c) 1300 cm²
 - d) 1500 cm²
5. Given the volume of a cuboid is 275 cm³ and the base area is 25 cm², what is the height of the cuboid?
- a) 10 cm
 - b) 11 cm
 - c) 12 cm
 - d) 13 cm

Unit Summary

This unit explores how geometric principles can be applied to understand and manage menstruation, particularly in relation to menstrual products and cycle tracking. Geometry helps in designing menstrual products like pads and tampons, optimizing their shapes and sizes for comfort and functionality. Understanding the geometry of the female reproductive system also aids in medical education and healthcare practices. Additionally, this unit will delve into the cyclical nature of menstruation, which can be represented through geometric graphs. By utilizing geometric shapes and concepts such as circles and cycles, students can better visualize and analyze the menstrual cycle, track symptoms, and predict future cycles.

Learning Objectives

- Understand the geometric shapes and designs of menstrual products.
- Explore how geometry can aid in modeling the menstrual cycle.
- Develop skills in using geometric principles to solve real-life problems related to menstruation.

Glossary

1. Menstruation: The monthly discharge of blood and mucosal tissue from the inner lining of the uterus through the vagina.
2. Cycle: The recurring series of physiological changes in the female reproductive system, typically lasting around 28 days.
3. Geometric Shape: A defined form that can be measured, such as circles, squares, or triangles, often used to describe the design of menstrual products.
4. Radius: The distance from the center to the edge of a circle, important for understanding the dimensions of circular menstrual products.
5. Diameter: The distance across a circle through its center, which can influence the size of menstrual pads and cups.
6. Symmetry: A property where one side of a shape mirrors the other, relevant in the design of comfortable menstrual products.
7. Volume: The amount of space an object occupies, which is crucial when considering the capacity of menstrual cups.
8. Area: The measurement of the surface within a shape, used to calculate the absorbent area of pads and liners.
9. Graph: A visual representation of data, such as a graph of menstrual cycle patterns over time, used for tracking and analysis
10. Predictive Modeling: A mathematical process that uses geometric concepts to forecast future menstrual cycles based on past data.

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